1958

Some Hydrodynamical Aspects of Oscillating Symmetrical Bodies in Liquid Helium II.

Billy J. Good
Louisiana State University and Agricultural & Mechanical College

SOME HYDRODYNAMICAL ASPECTS OF OSCILLATING SYMMETRICAL BODIES IN LIQUID HELIUM II

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Physics

by
Billy J. Good
B.S., Arkansas State Teachers College, 1950
M.S., University of Arkansas, 1952
February, 1958

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Dr. J. M. Reynolds for his encouragement, guidance and supervision in this research.

He is also indebted to his wife for her aid and understanding during the course of this work.

He wishes to thank Mr. W. J. Schultis, Mr. R. Deck, Mr. Clyde Bergeron, Mr. S. A. Ali and other members of the low temperature group for their helpful suggestions and invaluable assistance.

TABLE OF CONTENTS

Chapter		Page
I	INTRODUCTION	1
II	APPARATUS	13
III	PROCEDURE	29
IV	DEVELOPMENT AND DISCUSSION OF THEORETICAL ASPECTS	34
V	DISCUSSION OF THE DATA OBTAINED FROM THE EXPERIMENT	41
	SELECTED BIBLIOGRAPHY	52
	APPENDIX I	54
	APPENDIX II	62
	APPENDIX III	67
	AUTOBIOGRAPHY	71

LIST OF FIGURES

Figure		Page
1	The Column Oscillator	14
2	The Suspension System	16
3	Block Diagram of Timing Circuit	23
4	Circuit Diagram of A.C. Amplifier and Pulse Shaper	24
5	Circuit Diagram of D.C. Amplifier and Pulse Shaper	25
6	Circuit Diagram of Filament Power Supply for the D.C. Amplifier	26
7	Circuit Diagram of the Gate Control Circuit	27
8	Theoretical (Uncorrected for Boundary) and Experimental Values of the Effective Inertia at Various Temperatures Obtained with the Column Oscillator	42
9	Theoretical (Corrected for Boundary) and Experimental Values of the Effective Inertia at Various Temperatures Obtained with the Column Oscillator	44
10	Values of the Effective Inertia due to the Superfluid Density and Experimental Values of the Effective Inertia Obtained at Various Temperatures with the Column Oscillator	45
11	Experimental Values (Uncorrected) of ρ_N/ρ Obtained with the Gauze Oscillator Compared to Data of Peshkov at Various Temperatures	48
12	Experimental Values (Corrected) of ρ_N/ρ Obtained with the Gauze Oscillator Compared to Data of Dash and Taylor at Various Temperatures	49
13	Experimental Determination of the Constant σ	51

LIST OF ILLUSTRATIONS

Plate		Page
I	The Cryostat with the Column Oscillator	18

ABSTRACT

Various investigators have determined the ratio of the density of the normal component to the total density (ρ_N/ρ) of liquid Helium II by measuring the moment of inertia of disk type oscillators rotating in liquid Helium. In the present experiment an oscillator made from a roll of Dacron gauze was found to be more efficient for the determination of ρ_N/ρ. However, the values of ρ_N/ρ obtained with the gauze oscillator did not agree with the previously reported values obtained using disk type oscillators. These discrepancies were too large to be accounted for with the usual penetration depth and viscous loss type corrections.

The concept of effective mass as postulated by G. G. Stokes for the motion of symmetrical bodies through incompressible, irrotational, non-viscous fluids was applied to the fibers of the gauze oscillator moving through the superfluid component of liquid Helium II. The contribution of this effective mass to the inertia, i.e. effective inertia, was obtained by normalizing the equation for ρ_N/ρ which contained effective inertia to the values of ρ_N/ρ obtained from second sound data at low temperatures (1.35°K). Subsequent values of ρ_N/ρ obtained using the effective inertia correction factor were in good agreement with recently reported values in the same temperature range.

A column oscillator was used to verify the concept of effective inertia. It was found that the contribution by the super-fluid component of Helium II to the effective inertia agreed with the values predicted by classical hydrodynamical considerations. There exists a

deviation however, between the observed and predicted values of effective inertia due to the normal component of Helium II.

I. INTRODUCTION

The properties and behavior of liquid helium have been the object of much experimental and theoretical research in the field of low temperature physics. One of the reasons for this interest is the existence of a two phase system and a phase transition in liquid helium. The first experimental evidence of this phase transition was found by Kamerlingh-Onnes[1] in 1911. Then in 1927 W. H. Keesom and his coworkers[2,3] verified the existence of the two phases and found that the phase transition occurred at 2.18°K. They designated the phase existing below 2.18°K as Helium II and the phase existing between 2.18°K and the boiling point (4.2°K) as Helium I. Of the two phases, Helium II has proved to be the more interesting because of its unusual properties.

The first experiments to determine the viscosity of Helium II by flow methods were done by Allen and Misener,[4] and by Kapitza.[5] Anomalous results were obtained however, and it was shown that the fluid did not flow like an ordinary liquid through narrow channels but appeared to be non-viscous. Other experiments to determine the viscosity of Helium II

---

[1] H. Kamerlingh-Onnes, Proc. Acad. Sci. Amsterdam 13, 1093 (1911).

[2] W. H. Keesom and M. Wolfke, Leiden. Comm. 190b. (1927).

[3] W. H. Keesom and M. Wolfke, Proc. Acad. Sci. Amsterdam 31, 81 (1928).

[4] J. F. Allen and A. D. Misener, Nature 141, 75 (1939).

[5] P. L. Kapitza, Nature 141, 74 (1938).

were designated by Wilhelm, Misner and Clark[6] and by Keesom and McWood.[7] These researchers tried to measure the viscosity by allowing a disk to oscillate in the liquid. However their results did not agree with those determined by the flow methods.

Tisza[8,9,10,11] in an attempt to explain these contradictory values for the viscosity obtained by the two different methods, postulated that part of the atoms of Helium II taking part in the flow method had "super-fluid" properties. To account for the experimental results he considered liquid Helium II as consisting of two interpenetrating fluids, each possessing its own inertia and velocity. The amount of each fluid existing at any one temperature was represented by an internal parameter, x, which would vary from 0 to 1 in the temperature range from 0°K to 2.18°K. Thus one fraction, represented by ρx, behaves like a normal fluid, possessing the properties of viscosity and the ability to exchange momentum, to be scattered by heat waves and to adhere to the walls of the container. The other fraction, represented by $1-\rho x$, has zero viscosity, zero temperature, zero entropy and is called the super-fluid component. This two fluid model has indeed provided a satisfactory explanation for many of the unusual properties of Helium II.

---

[6]J. O. Wilhelm, A. D. Misener, and A. R. Clark, Proc. Roy. Soc. (London) A151, 342 (1935).

[7]W. H. Keesom and G. MacWood, Physica 5, 737 (1938).

[8]L. Tisza, Nature 141, 913 (1938).

[9]L. Tisza, Compt. rend. (Paris) 207, 1035 (1938).

[10]Ibid., p.1186.

[11]L. Tisza, J. phys. radium 1, 165, 350 (1940).

The theoretical background of the two fluid model has been built up by two entirely different approaches. London[12,13,14] has described the separation into two fluid components as a Bose-Einstein condensation. Landau[15] however, considered helium as background fluid in which excitations move. As a result of his study of quantum hydrodynamics, he proposed that the excitations be of two kinds. Those with the lowest energy he called phonons or quantized sound waves, whose energy E, is equal to pc, where p is their momentum and c is the velocity of sound. The excitations of higher energy, separated from the phonons by an energy gap Δ, were designated rotons or quantized vortices. These excitations can move about, exchange momentum, possess the properties of a viscous fluid, and in general behave like a normal fluid. The background fluid has the properties of the familiar super-fluid. But in either approach, i.e. the Bose-Einstein statistical method or the quantum hydrodynamics method, Helium II possesses the properties of two distinct fluids, one a super-fluid and the other a normal fluid.

To experimentally verify the various aspects of the two fluid model various experiments were designed to measure the respective densities of the normal and super-fluid components of Helium II. The first results were obtained by Andronikashvilli[16] who measured the ratio of

---

[12] F. London, Nature 141, 643 (1938).

[13] F. London, Phys. Rev. 54, 947 (1938).

[14] F. London, J. Phys. Chem. 43, 49 (1939).

[15] L. Landau, J. Phys. U.S.S.R. 5, 71 (1941).

[16] E. L. Andronikashvilli, J. Phys. U.S.S.R. 10, 201 (1946).

ρ_N/ρ (normal fluid density/total fluid density) by determining the moment of inertia of a stack of closely spaced aluminum disks oscillating in liquid Helium II. The significance of the experiment was in the design of the oscillator. If Landau's[17] theoretical considerations were correct then it should be possible to construct a vessel so that the moment of inertia of the vessel, containing Helium II, would vary from a maximum value at the λ-point (2.18°K, the temperature of the phase transition) to a minimum value at 0°K. The moment of inertia would be that of the vessel plus that due to the normal component of Helium II which would move with the vessel. The oscillating vessel consisted of a pile of one hundred disks of thin aluminum foil separated by spacers. The thickness of these spacers was of the order of two times the penetration depth, P (where the penetration depth is usually defined as that distance from a surface at which the shearing force due to the viscous coupling has been reduced to 1/e of the value at the surface). P may be represented by

$$P = \sqrt{2\eta/\omega\rho}, \qquad \text{(I. 1)}$$

where η is the viscosity of the liquid, ω is the frequency of the oscillations and ρ is the density of the liquid. For a frequency of 2π sec^{-1}, P is 0.1 mm., hence the spacers were of the order of 0.2 mm.

The super-fluid component was assumed to have no effect on the rotation of the disks. The viscous component, due to the spacing of the disks, would ride with the rotating disks, hence effectively increasing their mass. This increase in mass would subsequently increase the moment

---

[17]Landau, op. cit., 5, 71.

the velocity of second sound by Peshkov,[23,24] Band and Meyer[25] and de Klerk, Hudson and Pellam.[26] The values of ρ_N/ρ obtained by their calculations plotted versus temperature also showed that if the above equation is to hold then σ must indeed to a variable. They found that σ has a value of 4 for °K<0.6, approximately 13 in the range 0.6°K to 0.8°K and approximately 5.3 in the range 0.8°K to 2.2°K.

In the temperature range of 0°K to 1°K the values of ρ_N/ρ calculated from second sound measurements are considered more reliable than values measured by other means because good values can be obtained for the parameters S and C, where S is the entropy and C is the specific heat in the second sound equation,

$$U_{II}^2 = x(1-x)\, S^2\, T/C. \qquad \text{(I. 3)}$$

Here U_{II}^2 is the velocity squared of second sound and x is ρ_N/ρ . At higher temperatures the values of S and C are knows to an accuracy of only 10%. Therefore the data obtained in the upper temperature range are questionable. However since at 1.5°K only about 10% of the total density is ρ_N and this decreases rapidly as the temperature is lowered, the experimental errors involved in measurements with previous disk systems become quite large. This leaves a temperature range (roughly from 1°K to 1.5°K) where the experimental values of ρ_N/ρ are not very well known. Thus it would seem that if more work on this temperature range could lead

---

[23]V. P. Peshkov, J. Phys. U.S.S.R. 10, 389 (1946).

[24]V. P. Peshkov, J. Exptl. and Theoret. Phys. U.S.S.R. 18, 951 (1948).

[25]W. Band and L. Meyer, Phys. Rev. 74, 386 (1948).

[26]D. de Klerk, R. P. Hudson, and J. Pellam, Phys. Rev. 89, 662 (1953).

to better values of ρ_N/ρ it would be quite significant. Since it is theoretically possible to obtain these values by direct measurement methods, the main problem becomes the design of a more efficient oscillator, where efficiency refers to the change in the period of the oscillator due to its change in inertia. From the simple equation of a torsional oscillator,

$$\Theta_o = K\ (I_o)^{1/2} \tag{I. 4}$$

where Θ_o is the period and I_o is the moment of inertia of the oscillator in vacuum, it is seen that

$$\Delta\theta = (\Delta I)/2\ (I_o)^{1/2}. \tag{I. 5}$$

From this equation it follows that I_o should be made as small as possible, the lower limit depending on the practicability of limiting the angular velocity of the oscillator to less than the critical velocity. (Above this velocity normal fluid is generated.)

To make I_o small necessitates the construction of a light weight framework whose members are rigid enough to maintain their relative dimensions and are durable enough to rotate about an axis of symmetry as a rigid body. One possibility would be a three dimensional mesh with spacings less than two times the penetration depth. The material of the mesh must of course have a density greater than the displaced fluid (or the system could be weighted down) to prevent the oscillator from floating in the liquid helium.

As an approximation of this idealized oscillator the author, et al, used a strip of Dacron gauze rolled securely onto a Lucite spindle. This oscillator differs from the ideal in that the radial spacing of the fibers are much closer than two times the penetration depth. This necessitated of inertia of the vessel. This increase in moment of inertia would then cause the period of the oscillator to increase. Therefore, by measuring the variation in the period of oscillation with change in temperature, Andronikashvilli was able to calculate the variation of the total moment of inertia of the oscillator and thereby obtain the relative amount of the viscous, or normal component present at any temperature.

More recently Hollis-Hallett[18,19] refined and repeated the disk type oscillator experiment. He re-solved and corrected the Navier-Stokes equation as applied to the motion of a viscous fluid in the neighborhood of an oscillating disk. The results of this experiment agreed with those of Andronikashvilli, with particularly good agreement in the region near the λ-point. As a result of these experiments it is now believed that the equation for the variation of ρ_N/ρ with temperature,

$$\rho_N/\rho = (T/T_\lambda)^\sigma, \qquad (I.\ 2)$$

cannot have a unique value for σ over the temperature range from 0°K to the λ-point as predicted by London[20,21] and Tisza[22] who suggested a single value of σ equal to 5.5.

Values of ρ_N/ρ have also been calculated by the measurement of

---

[19] A. C. Hollis-Hallett, Proc. Roy. Soc. (London) A210, 404 (1952).

[20] F. London, Revs. Modern Phys. 17, 310 (1945).

[21] F. London, Report of International Conference of Physical Society (London) 2, 1 (1946).

[22] L. Tisza, Phys. Rev. 72, 838 (1947).

more layers of gauze per unit volume of oscillator, thus decreasing the ratio $\Delta I/(I_o)^{1/2}$. However, the percentage efficiency, $\Delta\theta/\theta_o$, was 23.00% where that found by Andronikashvilli[27] was 18.50% and that of Hollis-Hallet[28,29] was 13.36%, while that found by Dash and Taylor[30] was 12.20%.

Using this oscillator measurements have been made in the temperature range from 1.4°K to 2.18°K. The values of ρ_N/ρ calculated from these measurements were much larger than the values reported from the disk type oscillators. Correction factors for the change in the penetration depth with change of ρ_N/ρ in temperature were unable to reduce the observed values of ρ_N/ρ to previously reported values. This was particularly true at low temperature (1.4°K) where the values of ρ_N/ρ obtained were 100% greater than the values of ρ_N/ρ calculated from the data of disk type oscillators. These discrepancies led to the proposal by J. M. Reynolds[31] that the gauze oscillator should have added to its moment of inertia an effective moment of inertia due to the acceleration of the individual fibers of the oscillator in Helium II.

The geometrical differences between the disk and gauze systems are such that the solutions of the hydrodynamical equations necessary to describe the respective oscillators yield additional and significant

---

[27] Andronikashvilli, op. cit., 10, 201

[28] Hollis-Hallett, Proc. Phys. Soc. A63, 1367.

[29] Hollis-Hallett, Proc. Roy. Soc. A210, 404.

[30] J. G. Dash and R. Dean Taylor, Phys. Rev. 105, 7 (1957).

[31] Private communication.

information. The equation of motion for both oscillating systems can be represented by the differential equation,

$$I\ddot{\gamma} + L\dot{\gamma} + K\gamma = 0 \qquad (I.\ 6)$$

which has a solution,

$$\gamma = \phi e^{\beta t}. \qquad (I.\ 7)$$

γ is the angular displacement from the rest position and ϕ is the angular displacement of the system at time, $t = 0$. $\beta = -\Delta\omega + i\omega$, where Δ is the logarithmic decrement and $\omega = 2\pi/\theta$. $L\dot{\gamma}$ is the damping couple on the system due to the viscous forces of the fluid and the inherent damping in the suspension system. In order to evaluate the term $L\dot{\gamma}$ in the above equation it is necessary to solve the Navier-Stokes equation (1) to obtain the velocity distribution within the fluid, (2) to calculate from the velocity gradient at the surfaces of the oscillator the damping couple due to the viscosity of the fluid, and (3) to deduce equations from which the viscosity and density of the fluid may be determined from the observed logarithmic decrement and period of the oscillations. The solution of the Navier-Stokes equation for the two oscillators should lead to essentially the same results because all of the normal fluid between the disks or between the fibers will be carried with its respective oscillator.

For the disk system Hollis-Hallett[32,33] and Dash and Taylor[34] have

---

[32]Hollis-Hallett, Proc. Phys. Soc. A63, 1367.

[33]Hollis-Hallett, Proc. Roy. Soc. A210, 404.

[34]Dash and Taylor, op. cit., 105, 7.

solved the Navier-Stokes equation to obtain the above information. The solutions were possible because the disk system could be represented in the familiar cylindrical coordinate system. In the gauze system however, the Navier-Stokes equation would have to be written in a coordinate system or systems in which the individual fibers of the gauze could be represented. Such a solution has not been attempted by the author, but the system could presumably be represented by the union of three separate systems; (1) concentric annular rings evenly spaced in the radial direction (these spacings being determined by how tightly alternate layers of gauze are wrapped) and evenly spaced along the axis of rotation; (2) spokes radiating from the axis of rotation caused by the fibers of one layer being in contact with the fibers in an adjacent layer (spacing unknown); (3) cylindrical rods orientated co-axially with the axis of rotation, spaced evenly on a family of concentric circles (which are evenly spaced), the spacings being determined by the spacing of the fiber in the gauze and the tightness of the wrapping. Of these three systems it is seen that the latter two cause the super-fluid component of Helium II to undergo a displacement due to the motion of the fibers. This displacement of the super-fluid component is the most significant difference between the disk and gauze type oscillator. The design of the disk minimizes the displacement of super-fluid due to motion of the oscillator, whereas in the gauze this displacement is inherent.

The equation of motion of a body in an incompressible perfect fluid has been solved by Stokes.[35] Lee, Huang, and Yang[36] have shown that the

---

[35]G. G. Stokes, Trans. Com. Phil. Soc. 8, 105 (1843).

[36]T. D. Lee, K. Huang, and C. N. Yang, Phys. Rev., 106, 1135 (1957).

motion of super-fluid is irrotational, hence its velocity can be obtained from a velocity potential. Furthermore, since at constant temperature the density (ρ_s) of super-fluid is constant, the equation of continuity for a perfect fluid and super-fluid should be the same. Therefore, using the velocity solutions of the equation of continuity, the kinetic energy of the fluid due to the motion of a body in the fluid can be evaluated.

For the motion of a sphere in a perfect fluid Page[37] shows that this kinetic energy is $1/4\ M'V^2$, where M' is the mass of the liquid displaced by the sphere. Hence the presence of the liquid is equivalent to an addition of one half the mass of the liquid displaced. Therefore a force applied to the body in a perfect fluid must increase the kinetic energy of the liquid as well as that of the body. It is further seen that there is no resistance to a body moving through the fluid with constant velocity. The fluid moves from the front to the back of the moving body without exerting any resulting force on it. If, however, the body is caused to accelerate the applied force F, must overcome a loss of energy to the fluid. The applied force must be

$$F = (M + \alpha M')\ dV/dt \qquad (I.\ 8)$$

where M' is the mass of the displaced fluid and α is a geometrical factor. In the case of a sphere α is 1/2 and for a cylinder α is 1.

It follows then that the inertia of the gauze in Helium II will be increased by the addition of this effective mass term. This inertia is a function of some geometrical factor α, a constant, and s, a variable with respect to temperature. All the periods of oscillation of the gauze

---

[37]L. Page, Introduction to Theoretical Physics (Third Edition; New York: D. Van Nostrand Company, Inc., 1952), p. 245.

below the λ-point will be increased by this increase in inertia, hence must have this amount subtracted to obtain the correct values of ρ_n/ρ. Unfortunately α cannot be calculated due to the inability to represent the gauze in any known mathematical framework. However, a value for was deduced by normalizing the observed values of ρ_n/ρ at the lowest temperature to the values of ρ_n/ρ obtained from second sound data.

To verify the validity of the above concept of effective inertia, an oscillator for which the parameter α can be calculated has been made. It consists of four hollow Lucite rods affixed equidistant between two thin Lucite disks. The parameter was calculated by Deck and Reynolds[38] to have a value of 1. Using this value the theoretical and the observed values of the effective inertia agreed to within 0.3% at 1.4°K. At higher temperatures the deviation becomes approximately 20%. This difference is always such that the observed inertia is greater than the theoretical. This discrepancy is believed to be caused by some contributions of the normal component of Helium II since the error increases with the percentage increase of the normal fluid. Apparently the contribution of the normal fluid to the effective inertia is a strong function of the degree of perfection of the normal component. Since the normal fluid moves with the gauze oscillator it will not contribute to the effective inertia and thus the above discrepancy at higher temperatures will not appear in this system.

---

[38]Private communication.

II. APPARATUS

The gauze oscillator used in this experiment was made from Dupont Dacron gauze. The sheet of gauze was cut into seven strips, 1 by 72 inches. These strips were wound onto a spindle machined from a solid Lucite rod. The spindle was 3/8 inch in diameter and 1 inch long. It had a 1/16 inch hole drilled 1/8 inch deep in one end and the remaining part of the rod was drilled out to a diameter of 1/4 inch. The end of the first strip was glued to the spindle with Duco cement and allowed to dry. The layers were tightly wound onto the spindle in such a manner that the ends of successive strips overlapped for two or three turns. Thus the finished roll behaved as if it were rolled from one continuous strip of gauze. The end of the last strip was sewed with Nylon thread to the body of the roll of gauze. The finished roll had a diameter of 1-1/2 inches.

The other oscillator used consisted of four Lucite columns symmetrically placed between two thin Lucite disks. (Figure 1). The columns were machined from 1/4 inch solid Lucite rods. They were drilled out with a number one drill to a depth of 1-31/32 inches and cut off leaving a closed end 1/32 inches thick at one end. Two Lucite disks were machined down on a face plate to a thickness of 0.01 inches with an overall diameter of 1-1/2 inches. A 3/32 inch hole was drilled into the center of each disk while it was on the face plate. This assured a center for the oscillator. A jig was constructed to hold these pieces in place while they were being assembled. The jig was a solid brass rod which had been turned down and faced off in a lathe. While the piece was still in the lathe a 1/4 inch hole was drilled in its center and a 1.38 inch circle was

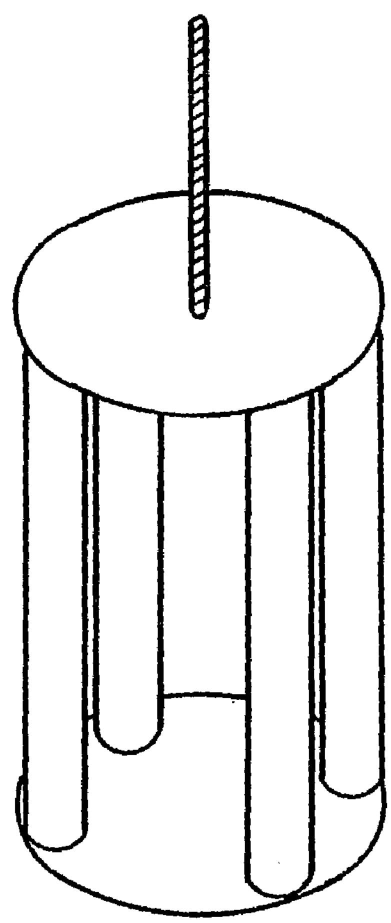


Figure 1

inscribed around this center. Four holes were laid off and drilled equidistant around this circle. The oscillator was assembled by placing the four Lucite columns in these four holes, a 1/4 inch steel rod which had a 3/32 inch diameter tip on one end into the center hole and centering a disk on this tip. The center rod floated in the center hole allowing the disk to come into contact with the ends of the columns. The ends of the columns were coated with Ethylene dichloride and a slight pressure applied to the disk welded these pieces together. After allowing to dry for 24 hours this part of the oscillator was removed from the jig and held in place by applying a coat of silocone vacuum grease to the disk and pressing it onto the machine flat. The remaining disk was centered onto the positioned columns and welded on as in the previous manner. The weld between the open end of the columns and the disk was reinforced by several applications of a solution of Lucite dissolved in ethylene dichloride. This was to insure mechanical strength and a super fluid tight seal.

The suspension system, Figure 2, was designed to allow height adjustment to compensate for a change in the length of the suspension fiber and to allow a zero adjustment. The height adjustment (A) is a knurlled brass knob fitted onto a 1/8 inch brass rod that has the lower end threaded. This rod (B) passes through an O-ring seal (C) in the zero adjustment (D) which is tapered and ground-in to fit a glass column which houses the suspension fiber. Onto the end of rod (B) is screwed and soldered an adaptor (C) to hold one end of the suspension fiber. The adaptor (G) is machined from a 1/4 inch brass rod. It has a 3/16 inch hole drilled perpendicular to the axis and has a tapered hole down the center with the large end of the taper terminating in the side of the 3/16

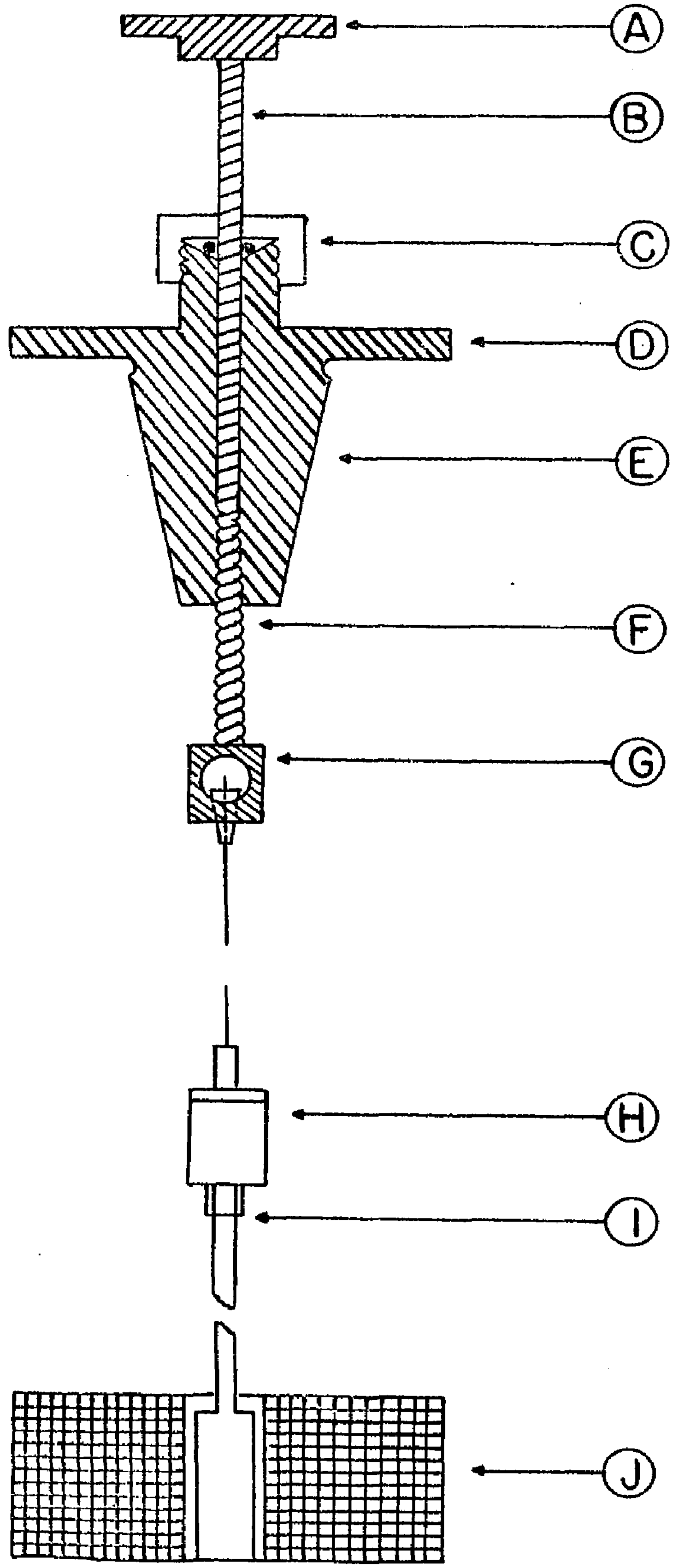


Figure 2

inch hole and the small end terminating at the lower side of the rod. A tapered plug fits into the tapered hole and has a small hole drilled co-axially in it. One end of the suspension fiber is threaded through this hole. The fiber is held down the side of the tapered plug and the plug is forced into the tapered hole. This securely holds the fiber and allows rapid assembly of the suspension system. The suspension fiber was a No. 46 nickel wire, 40 cm. in length. The lower end was built up by crimping a short section of nickel hypodermic needle onto the fiber. This end of the suspension fiber was glued with Duco cement into the cylinder that is the upper part of the mirror holder (H). This holder was cut and formed from 0.02 inch thick aluminum foil. It was rectangular in shape and had both ends formed into short cylinders. Onto the front of this holder was cemented a back-surfaced mirror with a piece of needle across its top, the needle length being equal to the width of the mirror. Into the bottom cylinder of the holder (H) was cemented a 3/32 by 18 inch glass rod (F). Onto the lower end of the glass rod the oscillator was centered and cemented with Duco cement.

The oscillator to be used was aligned onto the rod to minimize stirring of the liquid helium. This alignment was done after the suspension system was hanging free in the glass column of the cryostat. The oscillator was placed with its centering hole co-axially below the end of the glass rod stuck into the centering hole of the oscillator. The two were cemented in place with Duco cement and allowed to set while in this position.

The principle part of the cryostat, (Plate I) was a leveling platform. This was machined from a 1/4 inch brass sheet in the form of a 10 inch equilateral triangle. Leveling screws were placed at each corner.

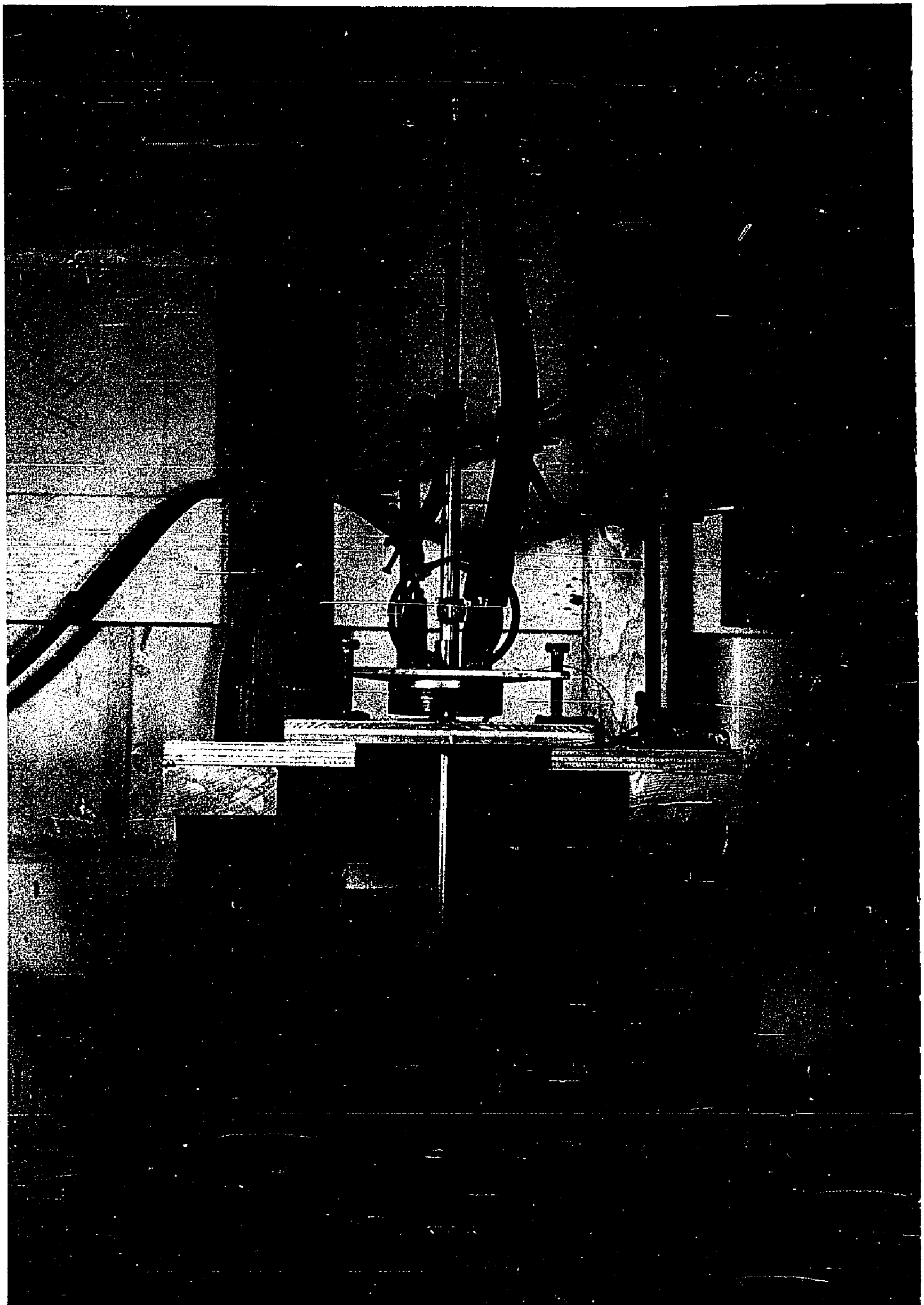


Plate I

To the top of this platform were soldered three flues. On the inside of the center flue were cut two O-ring grooves. These grooves, fitted with proper O-rings, formed the glass-to-metal seal between a glass column and the platform. This column was 18 inches long, 12 mm. O.D. and had one end beveled to facilitate easy insertion into the O-ring seal. Two inches above this beveled end, a short pyrex side arm was attached. This side arm terminated in a 1 inch flange which was ground flat. To this flat surface a 1 inch round optical flat was cemented with Glyptal cement forming the window for the suspension mirror. The upper end of the column was tapered to receive the tapered zero adjustment knob of the suspension system.

On either side of this column, a flue was soldered. During an experimental run a high pumping-rate mechanical vacuum pump was attached to one of these flues with a thick walled rubber vacuum hose. A section of this hose was constricted between the jaws of a modified machine vise. This vise was rebuilt to be controlled by a Selsyn motor-generator system through a two to one reduction gear drive. This allowed remote controlled changes in the size of the constriction of the tube. The size of this constriction determined the rate at which helium vapor was removed from the helium bath. In this way the pressure of the vapor above the liquid helium was regulated. Vapor pressures were measured with a mercury filled closed-arm manometer connected with a rubber vacuum hose to the other flue on the cryostat. An oil manometer containing Octoil "S" was also connected into this line to measure very low pressures. By referring these measured vapor pressures to a set of Mond vapor pressure tables, the temperature of the liquid helium was determined.

To minimize mechanically induced vibrations in the suspension system and to enhance the possibility of no shift in the zero setting of the optics, a pair of Helmholtz coils were used to start the oscillator swinging. These Helmholtz coils were wound on Phenolite forms of 4 inch diameter with approximately 20 turns of No. 28 enamel coated copper wire. The coils were centered about the mirror and separated from each other by a distance of 1-1/2 inches. Six volts D.C. were connected through d.p.d.t. reversing switch which would permit a deflection in either direction. This feature was extremely useful for stopping the oscillator system for zero adjustments and to determine if the system was being driven mechanically.

An isolation partition and radiation shield was attached to the underside of the leveling platform. This partition consisted of a perforated Pyrex flange sealed onto a 12 by 1 inch Pyrex tube. The small end was further reduced and attached to the extension of the center flue by a loose fitting aluminum support. Two thick felt washers slightly greater in diameter than the inner diameter of the helium flask were sewed to the perforated flange. An aluminum washer was placed on the top of the felt washers and acted as a radiation shield. This partition helped to remove impurities from that part of the liquid helium in contact with the oscillator during an experimental run.

A liquid helium flask was attached to the underside of the platform with an O-ring seal. The flask had a 84 cm. O.D. and a 70 cm. I.D. and was 55 cm. deep. A cork spacer taped to the bottom of the flask permitted it to be supported by the nitrogen flask. The nitrogen flask was held in place by a platform which was attached to the leveling platform.

This arrangement allowed the entire cryostat to be moved as a single rigid unit.

A dual purpose optical system was constructed to measure oscillator periods and amplitudes simultaneously. Two galvanometer light sources were mounted with a galvanometer scale on a movable carriage. The carriage was track mounted on a heavy frame which could be raised or lowered. The carriage itself could be adjusted horizontally with a screw. The lights were operated on separate DC sources to help eliminate noise in the timing circuit. The light used for amplitude measurements had the image of its hairline focused by a 1 meter lens onto the scale affixed to the movable carriage. An adjustable slit was mounted in front of the other light and its image was focused onto the slit system of a photomultiplier tube by means of the same 1 meter lens. This slit system of the photomultiplier tube was made by mounting a pair of razor blades at each end of a machined aluminum cylinder. The inside of the cylinder was painted with Aquadag to minimize reflected light. The slits were adjusted by inserting a feeler gauge between both sets of slits at the same time and moving the blades together for a tight fit. This spaced both sets of slits and aligned one set with the other in one operation. This slit system was then fitted tightly into a cylinder placed before a window of the photomultiplier tube housing. The complete assembly was light tight through the slit system. The slits were adjusted to 0.05 cm. for the collection of most of the experimental data and the image of the slit from the light source had a maximum width of 0.1 cm.

A timing system was designed to count and to time the periods of the oscillations of the oscillators used in these experiments. The mirror attached to the suspension system of the oscillator reflected the image of the illuminated adjustable slit onto the slit system of a photomultiplier tube as described above. As the image of the slit traversed the photomultiplier slit system the resulting pulse had the shape and length of the intensity profile of the image moving relative to the fixed slit. Therefore the length of the pulse was a function of the velocity of the oscillator. However, since the photomultiplier system was not an integrating device, the amplitude of the pulse was not velocity dependent. Thus it was possible to time the period of the oscillations by a device that was activated by a predetermined voltage level of the leading edge of pulses less than 2 seconds in length. A block diagram of the timing system is shown in Figure 3. The length of pulses to be timed was extended in later experiments by using a suitable D.C. amplifier in place of the A.C. amplifier in the pulse shaper.

The negative pulses from the photomultiplier tube had an amplitude of the order of 0.1 volt which was a function of the intensity of the light source, slit width and dynode voltage on the tube. These pulses were amplified and inverted in the first three stages of the amplifier. The amplitude of the pulses at this point could be varied by a volume control, making it possible to fire a neon glow tube at a predetermined voltage level of the pulse. The pulse from the glow tube was then shaped and matched to the gate control circuit. The circuit diagram for the A.C. amplifier and pulse shaper are shown in Figure 4 and similar diagrams for the D.C. amplifier, pulse shaper and power supply are shown in Figures 5 and 6.

The gate control circuit was a modified, T-61, model 222 counter which was rewired to count the individual swings of the light across the photomultiplier tube, Figure 7. It was used in the following manner.

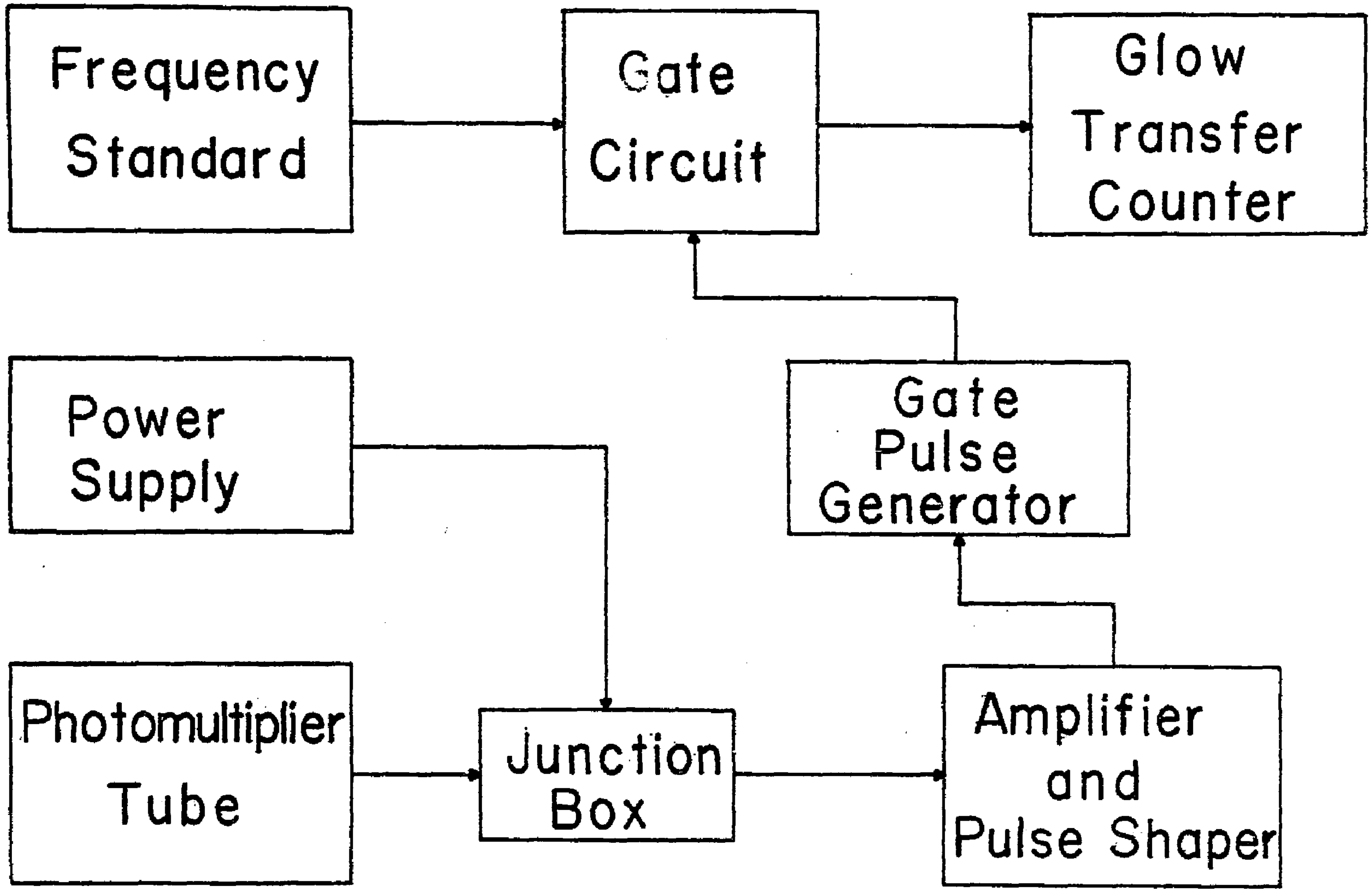

Frequency Standard
Gate Circuit
Glow Transfer Counter
Power Supply
Gate Pulse Generator
Photomultiplier Tube
Junction Box
Amplifier and Pulse Shaper

Figure 3

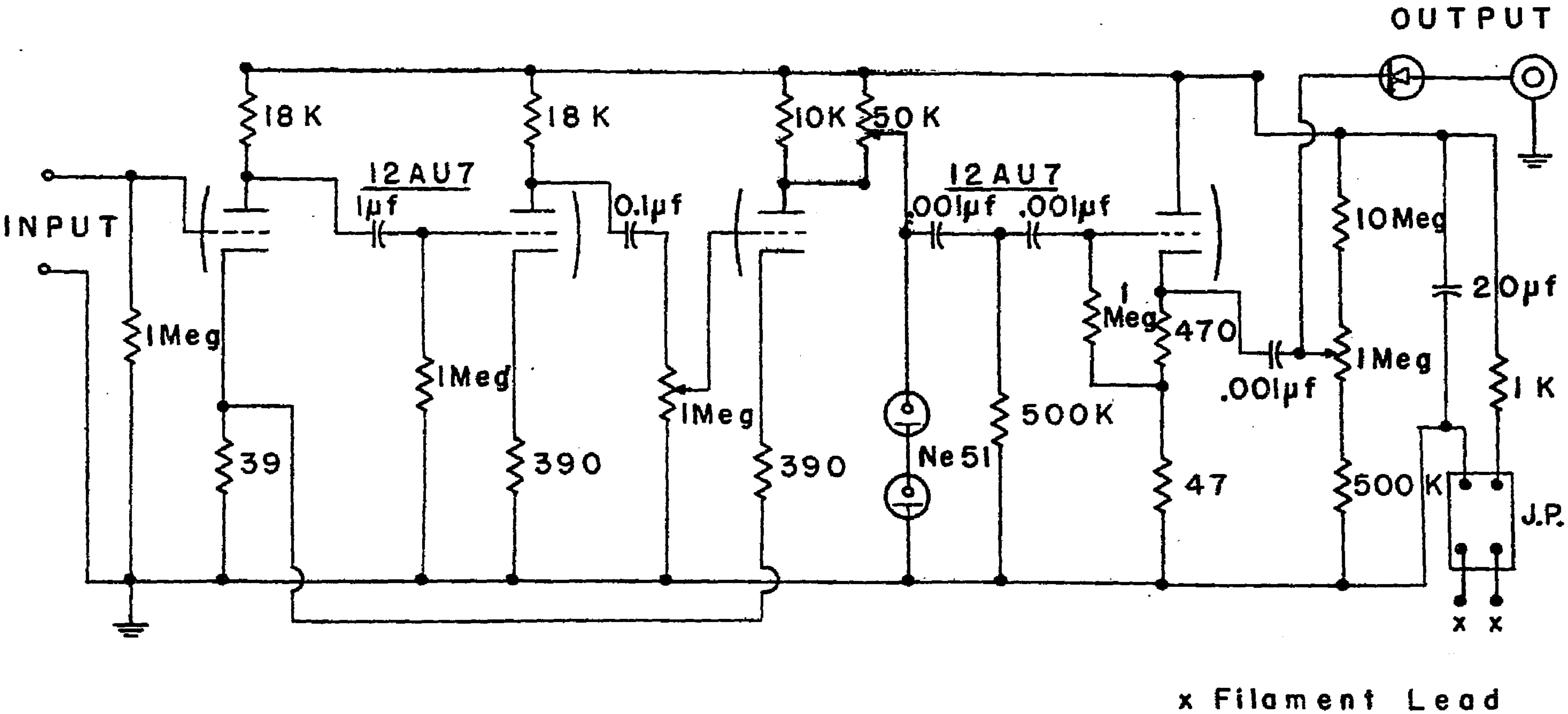
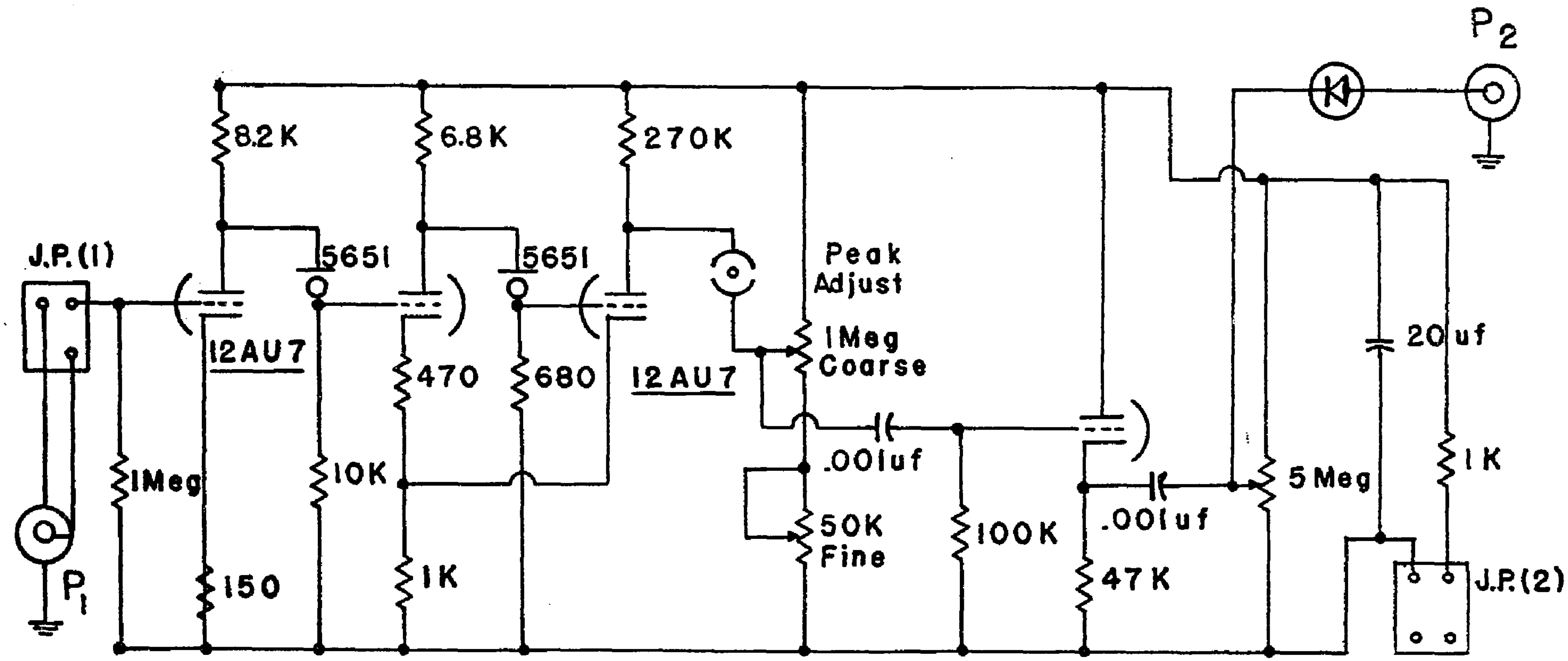

OUTPUT
INPUT
18 K
18 K
10K
50 K
12 AU7
12 AU7
1μf
0.1μf
.001μf
.001μf
1Meg
1Meg
1Meg
1Meg
10Meg
1 Meg
1Meg
39
390
390
Ne 51
500K
470
47
.001μf
500 K
20μf
1 K
J.P.
x
x
x Filament Lead

Figure 4



P2
8.2 K
6.8 K
270 K
J.P. (1)
5651
5651
Peak
Adjust
12AU7
1Meg
Coarse
470
680
12AU7
20 uf
1Meg
10K
.001uf
1 K
5 Meg
50K
Fine
100K
.001uf
P1
150
1K
47 K
J.P. (2)

Figure 5

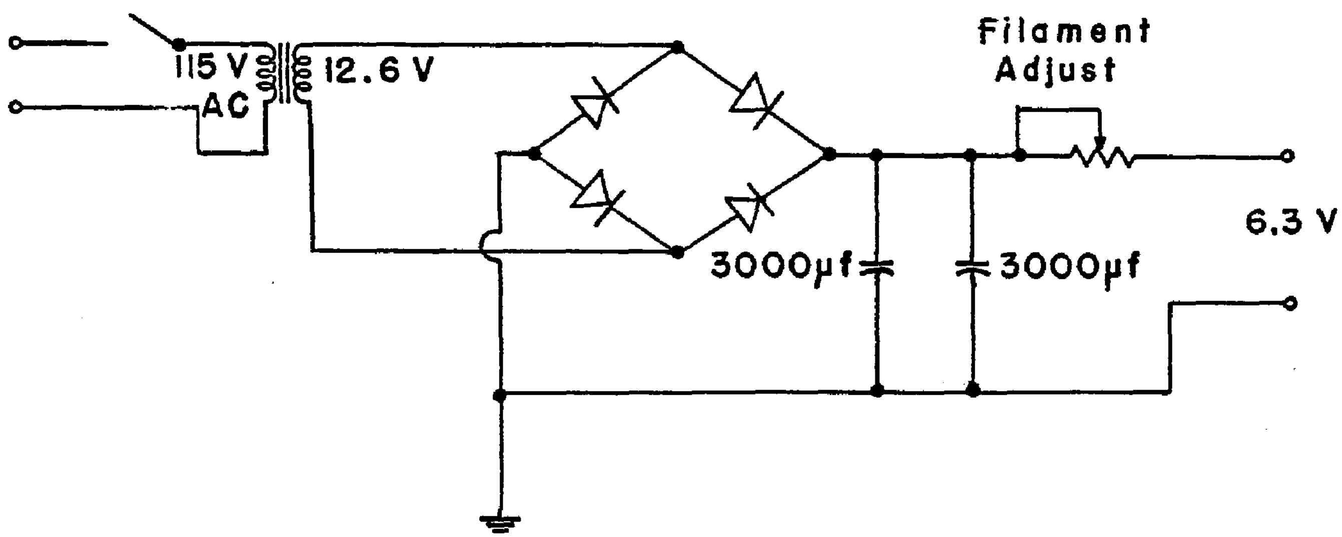


Figure 6

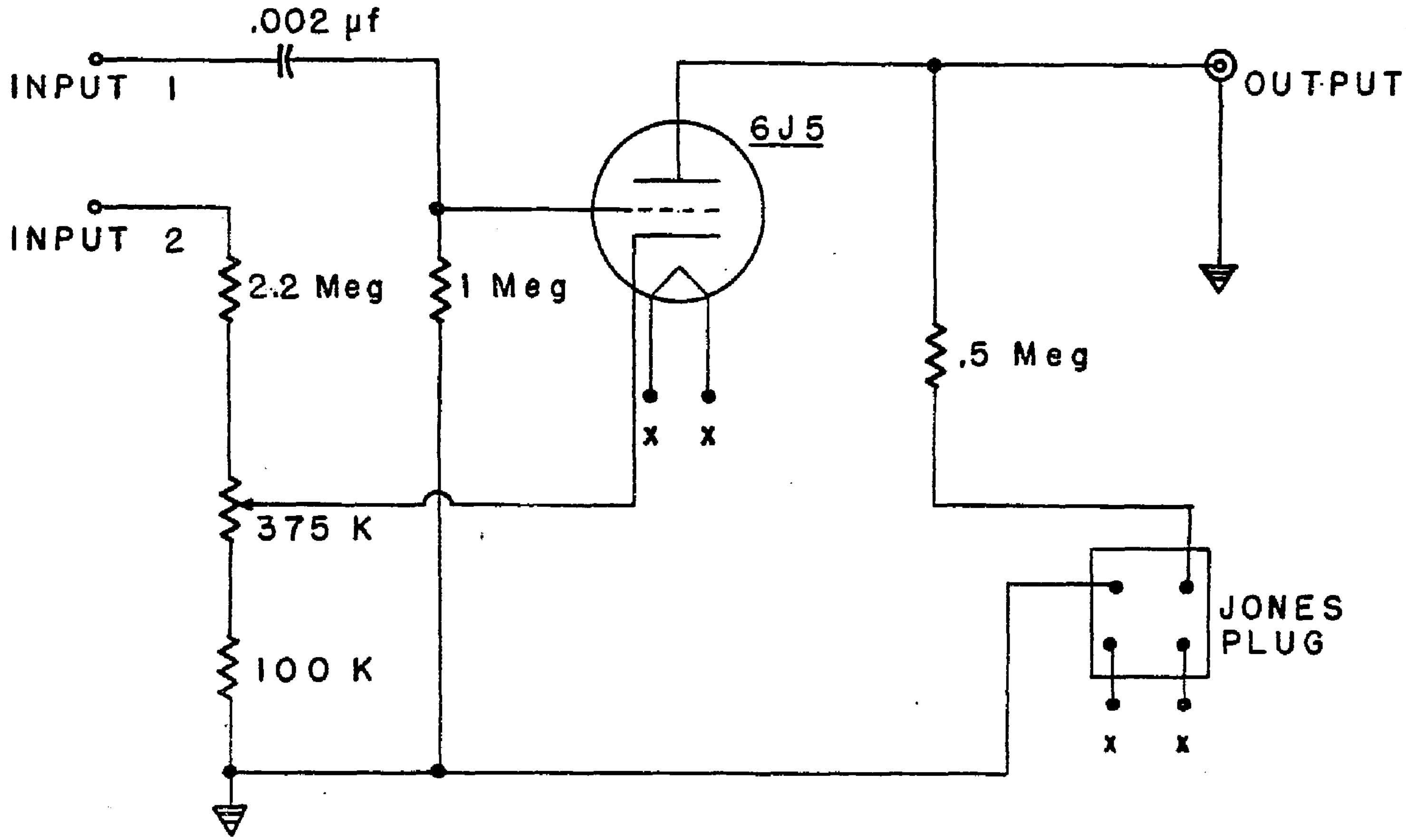

.002 µf
INPUT 1
INPUT 2
2.2 Meg
1 Meg
6J5
OUTPUT
.5 Meg
x x
375 K
100 K
JONES
PLUG
x x

Figure 7

The first swing produced a pulse which was stored in the counter, the second swing was stored and caused the last multivibrator to flip to the on position producing a positive swing to the gate signal. The next second, fourth or sixth preselected number of swings of the light were stored; the third, fifth or seventh pulse respectively flipped the multivibrator, turning off the gate signal. This produced a square gate signal of length equal to the time interval of one, three or seven complete periods of the oscillator circuit.

This gate signal removed the negative bias from a tube in the gate circuit. This tube had a standard pulse generator connected to its grid and a glow transfer counter (model 162A, Atomic Instrument Company) connected across its plate. When the bias was removed the counter counted the pulses from the standard pulse generator for the duration of the gating pulse.

The pulse generator was a "Low Frequency Standard" model 100D, Hewlett Packard. The pulses were produced by a crystal controlled oscillator. The crystal was maintained at a constant temperature which controlled the frequency to within 2 parts per million per week.

III. PROCEDURE

The oscillator to be used in a particular experimental run was attached to the suspension system as described earlier. It was visually checked while oscillating to determine if it was aligned co-axially. This alignment was necessary to prevent the outer surfaces of the gauze oscillator and the surfaces of the disk oscillator from stirring the liquid helium. After the oscillator was properly aligned the inner helium flask was put into position on the cryostat and held in place by the outer nitrogen flask. The inner flask was then sealed onto the platform with an O-ring seal. A charcoal trap was attached to one of the platform flues with a short section of thick wall vacuum hose. To the other flue an ion gauge, a thermocouple gauge, an oil manometer and a Welch Duo-seal vacuum pump were attached with rubber vacuum tubing.

The inner helium flask was evacuated and the pump hose was clamped off. The system was then checked for leaks by observing the rate of rise of the oil manometer. If the leaking rate was small the charcoal trap was reactivated by heating it with a gas burner while the system was being pumped out. After this outgassing process was completed the trap was allowed to cool to room temperature and the system was evacuated to the lowest pressure possible with the mechanical vacuum pump. The pump was then clamped off from the system, leaving the thermocouple gauge, the ion gauge, the cryostat and the charcoal trap as an integral unit. The charcoal trap was immersed in a Dewar flask filled with liquid nitrogen to further decrease the pressure of the system. The ultimate vacuum possible was at the point where the pumping rate became equal to the leaking rate. The pressure was determined with the thermocouple gauge until the range of this instrument was exceeded. At this point the thermocouple gauge was clamped off and the ion gauge circuit was switched on. The pressure finally attained in the system was determined to be 1×10^{-4} cm. of Hg.

The next step in the procedure was the alignment of the optical system. The optical frame was placed one meter in front of the cryostat window. The mirror of the suspension system was adjusted with the zero adjustment until its front surface was approximately parallel to the window surfaces. The optical frame and scale carriage were then adjusted until the reflected beam of the amplitude measuring light source was approximately centered on the perpendicular to the galvanometer scale. Then the period measuring light source was adjusted until the image of its slit fell on the slit system of the photomultiplier tube. The final adjustment of the optical system was then made by adjusting for maximum signal from the photomultiplier tube. The negative D.C. output pulse from the photomultiplier tube was displayed on a Dumont oscillascope. With the suspension mirror stationary, the photomultiplier tube and its light source were adjusted for maximum D.C. signal on the scope. At this point all adjusting devices were locked in place and this position was designated as the reference point or zero position of the suspension system.

The oscillator was rotated from the zero position by passing current through the Helmholtz coils, the amplitude of the oscillations depending on the duration of the current in the coils. With the oscillator in motion, the period timing system was set up. With the gate control circuit pulse off, the bias level was set until the amplitude of the one kilocycle signal from the standard pulse generator was reduced to some fraction of the amplitude necessary to trigger the glow transfer counter. Then with the gate control pulse on, the amplitude of the one kilocycle signal was checked to see if it was approximately the same fraction above the minimum amplitude necessary to trigger the glow transfer counter. When the signal was reduced by approximately the same fraction below cutoff with the gate control pulse off as it was above the cutoff point with the gate pulse on, the gate circuit was considered to be at optimum adjustment.

After the optical system was zeroed and the gating circuit adjusted, the A.C. amplifier pulse shaper circuit was adjusted so that a shaped pulse was produced each time the image of the lighted slit traversed the slit of the photomultiplier tube. If the system was properly aligned, the maximum of the pulse produced by the photomultiplier tube coincided with the zero position of the suspension system.

The maximum height of the amplified pulse at the final stage of the A.C. amplifier was set to correspond to the voltage level at which the Ne 51 neon tubes discharged. This caused the discharge of the Ne 51 neon tubes to occur at the peak of the input pulse. This adjustment was quite critical and the accuracy of the timing device depended upon how closely the peak of the pulse from the A.C. amplifier corresponded to the firing voltage of the neon tubes. This firing position was continuously monitored with the Dumont scope during a run and reset when necessary. When this adjustment was satisfactorily completed, the timing circuit was ready to time the periods of the oscillator.

An experimental run was made to determine the period and the logarithmic decrement of each of the oscillators at the minimum pressure obtainable in the system. The oscillator was deflected with the Helmholtz coils in such a manner as to cause a large amplitude on the scale. The timing circuit was started and individual periods were timed. Care was exercised to always start the clock with the light traversing the slit in the same direction. Simultaneously, amplitudes were recorded for several swings of the oscillator.

After the completion of such a vacuum run, dry helium gas was admitted into the cryostat raising the pressure to atmospheric pressure. The charcoal trap was removed and the thick walled vacuum hose to the primary mechanical vacuum pump was attached to the flue of the platform. The system was then flushed out with more dry helium, leaving a charge at atmospheric pressure in the cryostat. The hose was closed by the remote-controlled vise. Liquid nitrogen was admitted into the nitrogen flask to precool the system. The pressure was maintained at atmospheric pressure by letting in more helium as the system cooled. When no further decrease in pressure could be observed, the system was assumed to be precooled.

To perform the liquid helium experiments, the cryostat was first filled with liquid helium from a storage tank. A transfer tube from the storage tank was inserted through the other flue on the platform. When the transfer was complete, the ion gauge was left off and the vacuum hose was connected directly from the flue to the secondary vacuum system. This secondary vacuum system was left open to atmospheric pressure to allow the helium vapor to escape from the cryostat. The primary vacuum pump was started and the vise opened until a pressure decrease was noted in the mercury manometer of the secondary vacuum system. This system was then isolated from the atmosphere and the pressure in the cryostat regulated by the remote controlled vise.

The pressure was slowly reduced by opening the constriction of the vise until the lowest pressure obtainable was reached. At this low temperature the optical and electrical systems were reset as described above for a vacuum run. Amplitudes and periods for the oscillator being used were recorded for several different pressures corresponding to several different temperatures between the lowest temperature possible for the system and the λ-point. Usually if there was enough liquid helium remaining after data at the above temperatures were obtained, the pressure was reduced again. This second reduction in pressure usually resulted in a lower temperature in the system than was obtained from the first pressure reduction. Data at this lower temperature were taken and if necessary, data at higher temperatures were repeated.

Once the data for a given oscillator were taken the apparatus was allowed to warm up and the inner flask was removed and re-evacuated.

IV. DEVELOPMENT AND DISCUSSION OF THEORETICAL ASPECTS

The equations used for the determination of the ratio of the normal density of the total density (ρ_N/ρ) were derived from the classical equations for the motions of a damped harmonic oscillator. Torsional oscillators (such as the gauze oscillator designed for this experiment) which are subjected to velocity dependent damping forces can, in general, be described by the following differential equation,

$$I \frac{d^2\alpha}{dt^2} + L \frac{d\alpha}{dt} + K\alpha = 0, \qquad \text{(IV. 1)}$$

where I is the moment of inertia of the oscillator, $d^2\alpha/dt^2$ is the angular accelerations at any instant, L is a constant determined by the damping and K is a constant determined by the restoring torque of the suspension fiber. This equation can be rewritten as follows:

$$\ddot{\alpha} + 2K\dot{\alpha} + \omega^2\alpha = 0, \qquad \text{(IV. 2)}$$

by defining the constants in equation IV. 1 as

$$2K = L/I \text{ and } \omega^2 = K/I.$$

The general solution for an oscillator described by equation IV. 2 is

$$\alpha = C\, e^{-Kt} \sin\left(\sqrt{\omega^2 - K^2}\, t\right) \qquad \text{(IV. 3)}$$

if $K^2 < \omega^2$.

Equation IV. 3 shows that the system vibrates with a continually decreasing amplitude. With further simplification it can be shown that the periods, θ, between successive swings of the oscillator can be written as

$$\theta = 2\pi / \sqrt{\omega^2 - K^2}\,. \qquad \text{(IV. 4)}$$

If there is no damping (as in a vacuum) then $K = 0$ and

$$\theta_o = 2\pi/\omega \quad . \qquad \text{(IV. 5)}$$

It follows from equations IV. 3, IV. 4 and IV. 5 that $\alpha_n/\alpha_{n+1} = e^{\delta}$, where

$$\delta = 2K\pi / \sqrt{\omega^2 - K^2} \qquad \text{(IV. 6)}$$

and α_n is the amplitude of the n^{th} swing of the oscillator and δ is the logarithmic decrement. Therefore,

$$\theta_o/\theta = 2\pi / \sqrt{4\pi^2 + \delta^2} \qquad \text{(IV. 7)}$$

or

$$\theta_o^2 = (1 + \delta^2/4\pi^2)^{-1}\,\theta^2 \qquad \text{(IV. 8)}$$

Thus by measuring the change in amplitude of the damped system of the logarithmic decrement can be determined. The value of the logarithmic decrement and the period of the damped system can be used to determine the period of the undamped system from equation IV. 8. These corrected periods (hereafter defined as θ for simplicity) were then used in equation IV. 5 as

$$\theta = 2\pi\sqrt{I/K}$$

or simply as

$$\theta^2 = C\,I \quad . \qquad \text{(IV. 9)}$$

Thus a change in the moment of inertia of the system was observed as a change in the period of the oscillator. (The moment of inertia of the gauze oscillator, I_o, referred to the moment of inertia of the oscillator in vacuum.) In liquid Helium II below the λ-point the total moment of inertia as calculated by equation IV. 9 can be expressed as a sum of four different moments of inertia,

$$I = I_o + I_v + I_p + I_E \quad , \qquad \text{(IV. 10)}$$

where I_o is the moment of inertia of the system in vacuum, I_v is the moment of inertia of the volume of normal fluid contained between and moving the fibers of the oscillator, I_p is the moment of inertia due to the layer of liquid Helium clinging to the outer surface of the gauze oscillator and I_E is the effective moment of inertia contributed by the super-fluid component of liquid Helium that was displaced by motion of the individual fibers of the gauze oscillator.

The moment of inertia, I_p, can be obtained by calculating the moment of inertia due to a layer of liquid Helium of thickness P/2 on the outer surfaces of the oscillator. The penetration depth, P, is defined as

$$P = \sqrt{\theta \eta / \pi \rho_N} \qquad \text{(IV. 11)}$$

where η is the normal viscosity and ρ_N the normal density of liquid Helium. I_p can be expressed as shown in Appendix II, in terms of geometrical constants obtainable by calibration as

$$I_p = \rho_N \,(AP + BP^2) \quad . \qquad \text{(IV. 12)}$$

The moment of inertia, I_v, is obtainable by integrating $\int r^2\, dm$ over the volume of the oscillator and is just

$$I_v = 1/2 \,(\rho_N V)\, r^2 = V' \rho_N \quad . \qquad \text{(IV. 13)}$$

Here V is the volume of the oscillator excluding the volume of the fibers.

The equation for the effective moment of inertia, I_E, is

$$I_E = \sum_i \alpha V'' r_i^2 \rho_s = E \rho_s \qquad \text{(IV. 14)}$$

where α is a geometrical factor determined by the shape of the body. It has been shown to have a value of 1 for a cylindrical body rotating at a distance r about the axis of rotation.[39] V'' is the volume of the sum of the individual fibers that cause a displacement of the medium through

---

[39] W. Deck and J. M. Reynolds, private communication.

which they move. I_E is the effective inertia due to the sum of the effective masses of n fibers at a distance r_i from the axis of rotation.

Substituting equation IV. 10 into equation IV. 5 we obtain for the period of the gauze oscillator at any temperature below the λ-point,

$$\theta^2 = C\ (I_o + I_v + I_p + I_E) \quad . \qquad \text{(IV. 15)}$$

At the λ-point ρ_s equals zero, hence I_E equals zero and equation IV. 15 becomes

$$\theta_\lambda^2 = C\ (I_o + I_{v_\lambda} + I_{p_\lambda}) \quad . \qquad \text{(IV. 16)}$$

Combining equations IV. 15, IV. 16 and IV. 9 to eliminate the constant C and I_o we obtain:

$$\frac{\theta^2 - \theta_o^2}{\theta_\lambda^2 - \theta_o^2} = \frac{I_v + I_p + I_E}{I_{v_\lambda} + I_{p_\lambda}} \qquad \text{(IV. 17)}$$

Using the definition, $\rho = \rho_N + \rho_s$, and the usual assumption that $\rho_s = 0$ at the λ-point, equation IV. 17 becomes, after substituting for I_v, I_p, and I_E their defining equations, one obtains

$$\frac{\theta^2 - \theta_o^2}{\theta_\lambda^2 - \theta_o^2} = \frac{\rho_N\ (V' + AP + BP^2)}{\rho\,(V' + AP_\lambda + BP_\lambda^2)} + \frac{\rho_s\ (E)}{\rho\,(V' + AP_\lambda + BP_\lambda^2)}$$

$$= X\ \rho_N/\rho + Y\ \rho_s/\rho \quad . \qquad \text{(IV. 18)}$$

Substituting $\rho_s = \rho - \rho_N$ into equation IV. 18 and resolving for ρ_N/ρ we have

$$\frac{1}{(X - Y)}\left(\frac{\theta^2 - \theta_o^2}{\theta_\lambda^2 - \theta_o^2} - Y\right) = \frac{\rho_N}{\rho} \quad . \qquad \text{(IV. 19)}$$

From the definition of Y in equation IV. 18 it is seen that it is a constant whose value is determined by the effective inertia coefficient E, geometrical constants V', A and B and the penetration depth at the λ-point, P . X is a function of the penetration depth inherent in the I_p

term. ρ_N/ρnikashvilli[40] neglected this term in a similiar derivation for ρ_N/ρ and showed that the error resulting from this omission was less than 3%. Neglecting I_p in the above derivation for ρ_N/ρ would be the same as replacing X by 1 in equation IV. 19. Y would be a different constant dependent primarily on the value of the effective inertia coefficient, E.

The calibration of the gauze system to obtain the values for X and Y is discussed in Appendix II.

The column oscillator was used to experimentally verify the effective inertia concept as used in the treatment of the data obtained with the gauze oscillator. The effective inertia is the analogue of the effective or "virtual" mass as treated by Stokes.[41] He treated the case of symmetrical objects moving through non-viscous, irrotational, incompressible fluids and found that the effect on the motion of the object through the fluid by the presence of the fluid was the equivalent of adding to the mass of the object and additional mass. This additional mass was proportional to the mass of the displaced fluid. This effective mass is a consequence of the kinetic energy imparted to the fluid as well as to the body when a force is applied to the body. The kinetic energy imparted to the fluid is derivable from the components of the velocity of the fluid particles. The velocity components can be obtained from the hydrodynamical equation of continuity if the motion of the fluid is irrotational, non-viscous and incompressible.

---

[40]Andronikashvilli, <u>op</u>. <u>cit</u>., <u>10</u>, 201.

[41]Stokes, <u>op</u>. <u>cit</u>., <u>8</u>, 105.

Recently, Lee, Huang and Yang[42] have shown that the motion of the Helium fluid as a whole should exhibit the classical dynamics of an incompressible, irrotational, non-viscous fluid. The model presented by them is based upon quantum mechanical arguments describing the low temperature properties of a dilute Bose system of hard spheres. They have shown that the kinetic energy of the super-fluid flow can be represented by

$$\rho\int(\nabla\phi)^2\, dt \qquad \text{(IV. 20)}$$

in which dt represents an incremental box. $\nabla\phi$ is the gradient of the velocity potential ϕ. It is shown that the momentum of the super-fluid is equal to $\nabla\phi$ per particle. Therefore, from equation IV. 20, V_s equals $2\nabla\phi$ and from classical mechanics the curl of V_s is identically zero, therefore, V_s is irrotational. Therefore, it is assumed that the effective mass concept carries over exactly for the super-fluid component. Strong experimental evidence is presented to show that the normal component also exhibits a similiar effective mass term.

The details for the calculation of the effective mass, hence the effective inertia for the column oscillator have been worked out by Deck and Reynolds,[43] and their final equations are given below.

The effective inertia, I_E, due to an effective mass, M', at a distance, r_c, from the axis of rotation is

$$I_E = M'r_c^2 \quad . \qquad \text{(IV. 21)}$$

For the column oscillator the value of the effective mass, considering a bath of finite dimensions, is

---

[42] Lee, Huang, and Yang, op. cit., 106, 1135.

[43] Deck and Reynolds, op. cit.

$$M' = 4h \rho_s \pi a^2 (1 + 2 \xi_1) \tag{IV. 22}$$

where

$$\xi_1 = \frac{a^2 b^2}{(b^2 - r_c^2)^2} \tag{IV. 23}$$

and h is the height of the columns, a is their radium, r_c is the distance from the column to the axis of rotation and b is the radium of the Helium flask.

Substituting values for the above geometrical constants corrected for thermal contraction of Lucite into equation IV. 22 gives

$$I_E = 15.49 \rho_s + 154.75 \left\{ (a + P/2)^2 + 0.259 \ (a + P/2)^4 \right\} \rho_N \ . \tag{IV. 24}$$

Experimental values for I_E were obtained by substituting the measured period, θ, in vacuum and at various temperatures below the λ-point into equation IV. 9. Thus

$$\theta_o^2 = C \ (I_o) \tag{IV. 25}$$

and

$$\theta^2 = C \ (I_o + I_p + I_E). \tag{IV. 26}$$

Then

$$I_o \left(\frac{\theta^2 - \theta_o^2}{\theta_o^2} \right) - I_p = I_E \ . \tag{IV. 27}$$

The determination of the values for I_p for the column oscillator are discussed in Appendix I.

The agreement found between the theoretical and experimental values for the effective inertia is good, particularly at the lower temperatures.

V. DISCUSSION OF THE DATA OBTAINED FROM THE EXPERIMENT

1. Data of the Column Oscillator: Calculated and experimental data obtained using the column oscillator are tabulated in Appexdix III. Values for the effective inertia, I_E, were compared with three possible models representing the theoretical effective inertia. One model for the theoretical effective inertia assumes that the column oscillator is oscillating in a bath of liquid Helium II of infinite extent, thus eliminating any boundary effects due to a containing vessel. This equation for I_E is redefined to be

$$I_E = 154.75 \left\{ (a^2) \rho_s + (a + P/2)^2 \rho_N \right\} \quad . \qquad (V.\ 1)$$

The first term on the right side of the above equation gives the effective inertia due to the super-fluid density, and the second term the effective inertia due to the normal density.

The theoretical values for the effective inertia obtained using equation V. 1 were plotted and compared with experimental effective inertia, I_{Ex}, where

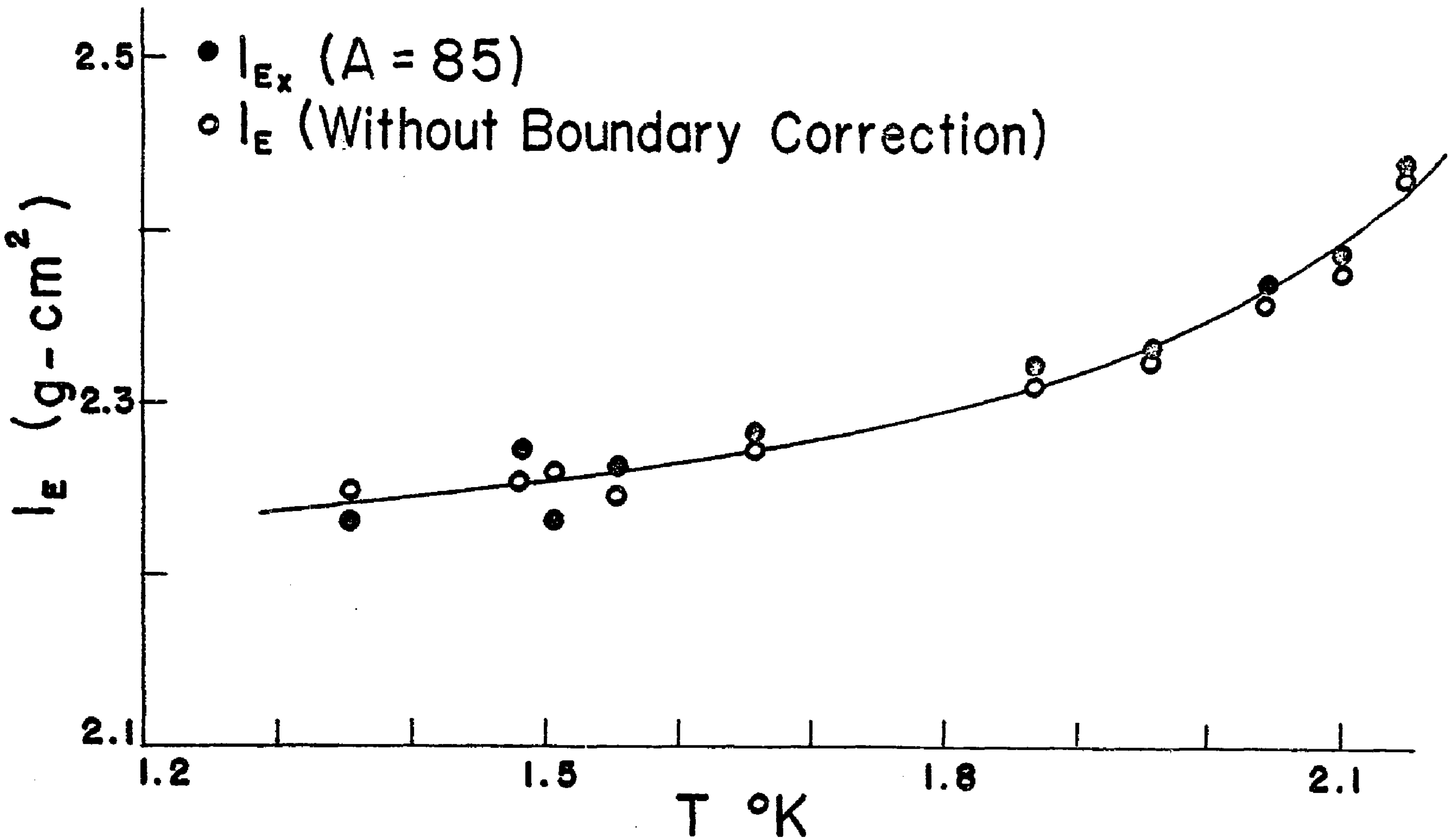
$$I_{Ex} = I_o \left(\theta^2 / \theta_o^2 - 1 \right) - \rho_N AP \quad . \qquad (V.\ 2)$$

A value of 85 was used for A in the above equation. The resulting curve is shown in Figure 8.

Another model for the theoretical effective inertia can be expressed as:

$$I_E = 15.490 \rho_s + 154.75 \rho_N \left\{ (a + P/2)^2 + 0.259 (a + P/2)^4 \right\} . \qquad (V.\ 3)$$

This model assumes a bath of liquid Helium II of finite dimensions and



2.5
2.3
2.1
1.2
1.5
1.8
2.1
T °K
I_E (g-cm^2)
● I_{Ex} (A = 85)
○ I_E (Without Boundary Correction)

Figure 8

equation V. 3 includes the correction for the boundary effect. In Figure 9 the experimental values of the effective inertia, I_{E_X}, as obtained using equation V. 2 and the theoretical values of I_E obtained using equation V. 3 were plotted

The models presented above assume that the flow lines originate and terminate on the surface of the liquid Helium film (of thickness P/2) which clings to the surface of the oscillator. This assumption has been shown by Prandtl[44] to be valid for the case where the Reynolds number is less than unity, i.e. $R < 1$, where R is defined as the usual way to be

$$R = V_M \, r \, \rho_N / \eta_N \quad . \qquad (V.\ 4)$$

V_M is the maximum velocity of fluid flow across the surface at some point on the oscillator and r is the distance from this point to the axis of rotation.

In the column oscillator experiment, however, the Reynolds number is much greater than one, hence it may be more correct to assume that the flow lines originate and terminate at the actual surface of the oscillator. This assumption would exclude any contribution to the effective inertia by the "clinging film" and equation V.3 would be modified to read

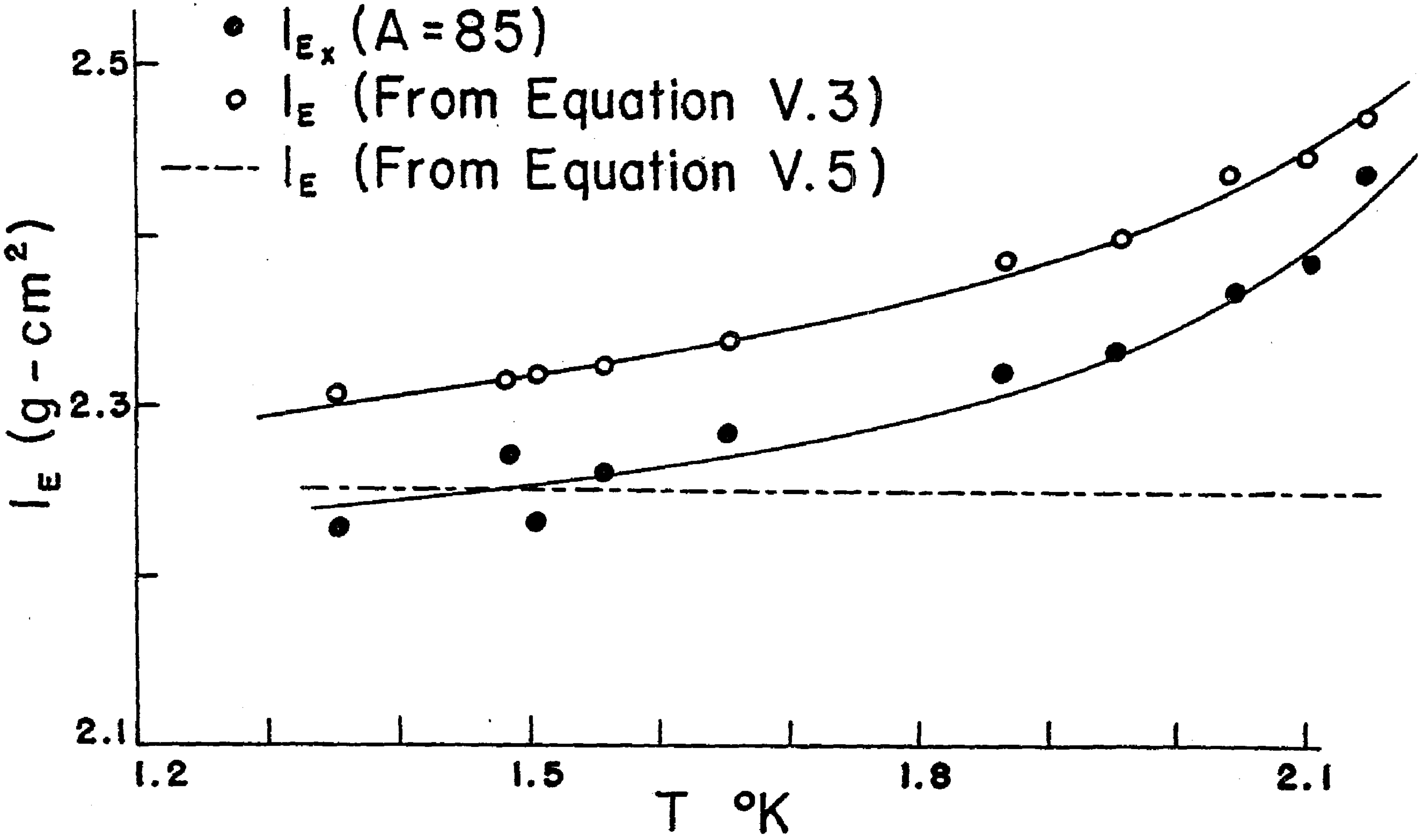
$$I_E = 15.49\,(\rho_s + \rho_N) = 15.49\,\rho \quad . \qquad (V.\ 5)$$

The curve representing this third model is shown in Figure 9, where a constant value of 0.145 for the total density, ρ , was used in equation V. 5.

Figure 10 is a plot of the effective inertia, I_s, due to the superfluid density, ρ_s, of liquid Helium II, and of the experimental values of the effective inertia, I_{E_X}, obtained using equation V. 2 and a value of

---

[44] L. Prandtl, Essentials of Fluid Dyamics, (New York: Hafner Publishing Company, 1952), p. 186.



I_{E_x} (A=85)
I_E (From Equation V.3)
I_E (From Equation V.5)
2.5
2.3
2.1
I_E (g-cm^2)
1.2
1.5
1.8
2.1
T °K

Figure 9

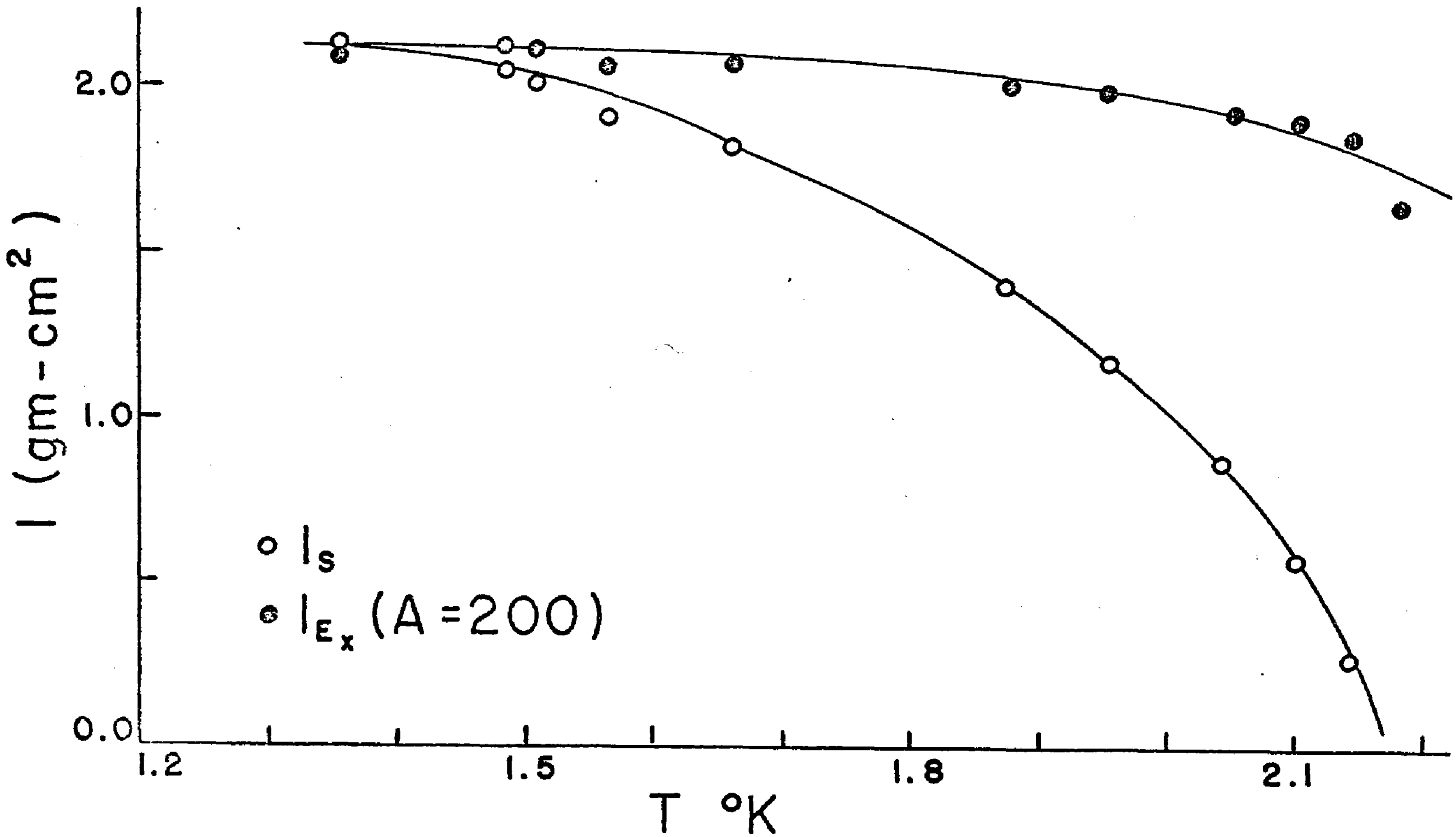


Figure 10

200 for A. It can be seem that the contribution to the effective inertia by ρ_s alone cannot account for the observed values of I_E at higher temperatures. However agreement is good at the lower temperatures where ρ_N becomes small.

It is concluded that the agreement between values of I_E and I_{Ex} as shown in Figure 8 is fortuitous since the agreement depends primarily on the assumptions that the "clinging film" contributes to the effective inertia and that there are no boundary effects. From Figures 9 and 10 it is concluded that the contribution by the super-fluid density, ρ_s, to the effective inertia is in excellent agreement with theory. This agreement excludes the possibility of a contribution to the effective inertia by the clinging layer. Therefore, the model represented by equation V. 5 is suggested as the best choice of the three possible models.

2. Data of the Gauze Oscillator: Two gauze oscillators, designated number 1 and number 2, were used to obtain the period and amplitude data tabulated in Appendix III. These oscillators had approximately the same dimensions but no attempt was made to make them identical. Since the data obtained from oscillator number 2 were taken over a greater range of temperature they were used from the evaluation of the geometrical factor Y in the equation redefined as:

$$\frac{1}{(X - Y)} \left(\frac{\theta^2 - \theta_o^2}{\theta_\lambda^2 - \theta_o^2} - Y \right) = \frac{\rho_N}{\rho} \quad . \qquad \text{(V. 4)}$$

To calculate ρ_N/ρ for these oscillators it was necessary to obtain a value for Y. This was done by substituting the value of $\theta^2 - \theta_o^2 / \theta_\lambda^2 - \theta_o^2$ obtained at the lowest temperature, T = 1.388°K, with oscillator number 1 and the value of ρ_N / ρ obtained from Peshkov's[45] second sound data at the same temperature into equation V. 4, assuming X = 1. The value of Y was found to be 0.1143.

A plot of the uncorrected data obtained with oscillator number 2 and the values of ρ_N / ρ obtained from Peshkov's second sound data is shown in Figure 11. It can be seen that the uncorrected data obtained with the gauze oscillator agrees with the second sound data near the λ-point but disagrees quite significantly at lower temperatures. The higher values of the uncorrected data at the lower temperatures are due to the contribution of the effective moment of inertia of the super-fluid component of liquid Helium II to the moment of inertia of the system. This increase in inertia subsequently increases the period, thus the ratio $\theta^2 - \theta_0^2 / \theta_\lambda^2 - \theta_0^2$. Since there is no contribution of the normal component of liquid Helium in the gauze oscillator the two curves should approach one another (as is shown in Figure 11) near the λ-point, where the percentage of super-fluid approaches zero.

Figure 12 shows a plot of the corrected data (using equation V. 4) obtained using oscillator number 1 and oscillator number 2. Also plotted on this same Figure is the data obtained by Dash and Taylor.[46] The agreement of the data obtained from the two gauze oscillators is justification for using this type oscillator. The agreement between the data obtained with the gauze oscillators and that of Dash and Taylor in the temperature range shown indicates the potentiality of such oscillators at

[45] V. P. Peshkov, J. Phys. U.S.S.R. 10, 389 (1946).

[46] Dash and Taylor, op. cit., 105, 7

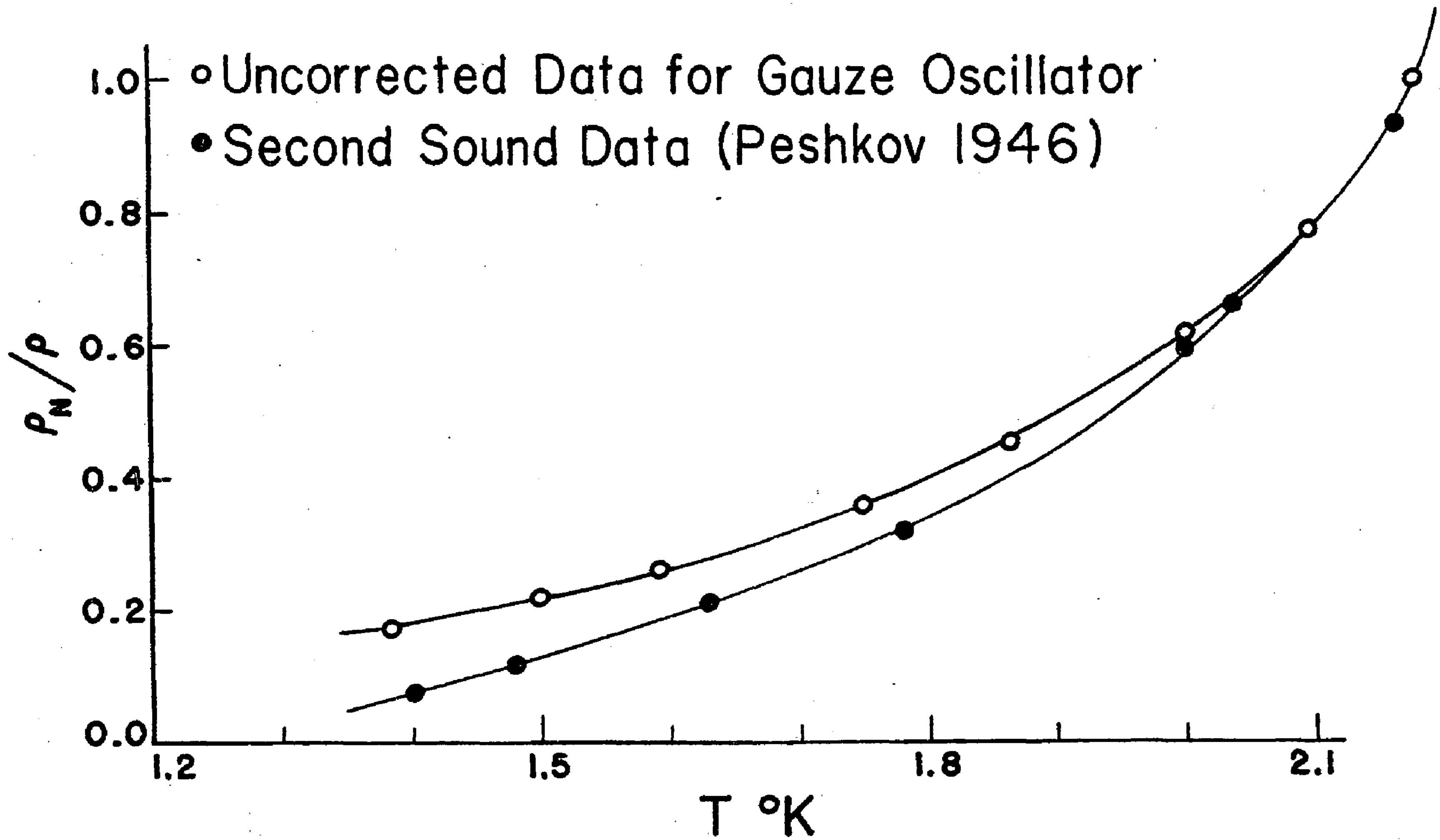
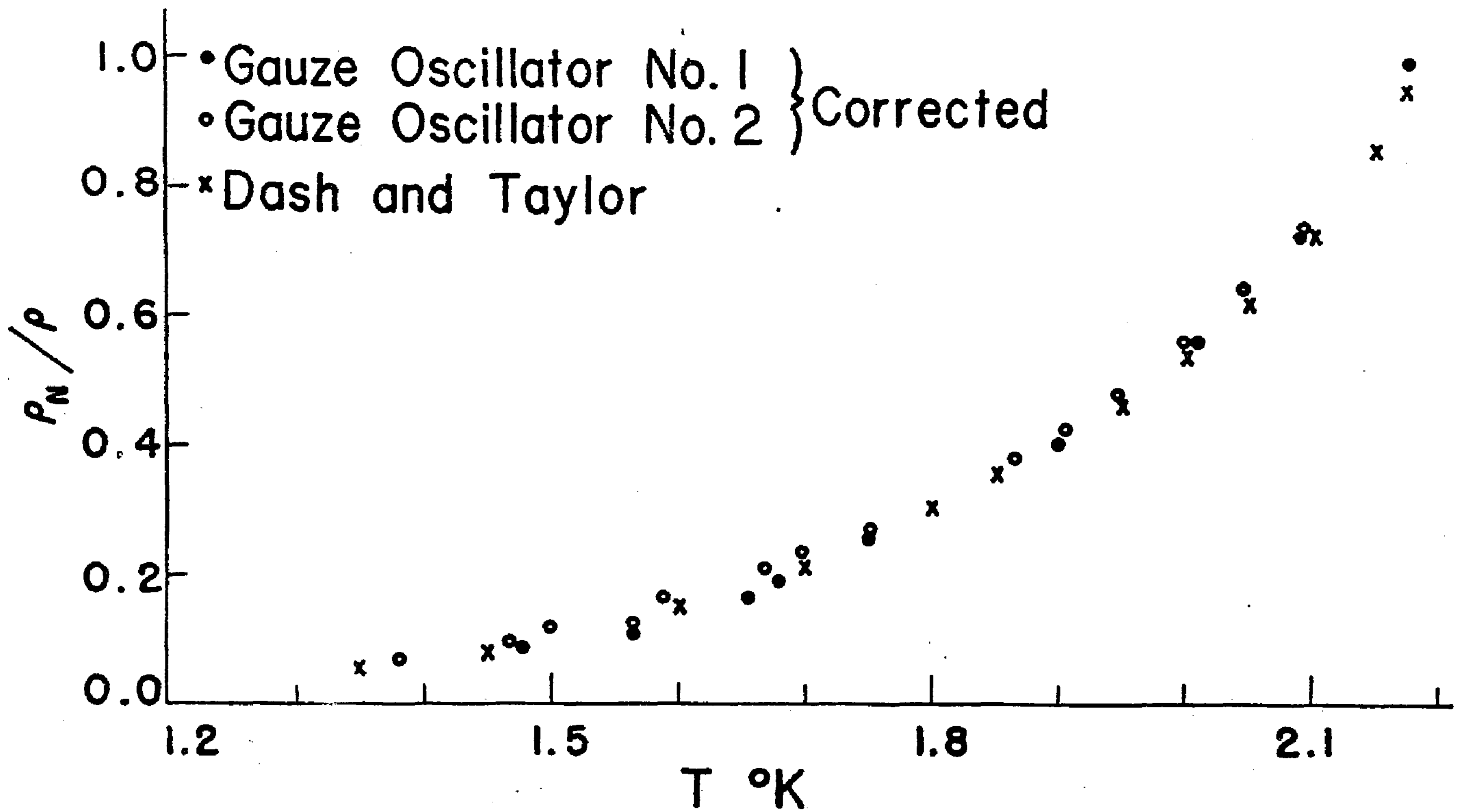

○ Uncorrected Data for Gauze Oscillator
● Second Sound Data (Peshkov 1946)
ρ_N/ρ
1.0
0.8
0.6
0.4
0.2
0.0
1.2
1.5
1.8
2.1
T °K

Figure 11



• Gauze Oscillator No. 1
∘ Gauze Oscillator No. 2
Corrected
× Dash and Taylor
1.0
0.8
0.6
0.4
0.2
0.0
ρN/ρ
1.2
1.5
1.8
2.1
T °K

Figure 12

lower temperatures. Here the efficiency of the gauze oscillator becomes significantly greater than that of the disk type oscillator.

Figure 13 is a plot of log ρ_N / ρ versus log T / T_λ, where T_λ is the temperature at the λ-point. The slope of this curve is the value of the exponent σ in the equation

$$\rho_N / \rho = (T/T_\lambda)^\sigma \quad . \qquad (V.\ 5)$$

This plot gives a value for σ of 5.7. This compares favorably to the value of 6.2 reported by Dash and Taylor.[47]

---

[47] Ibid.

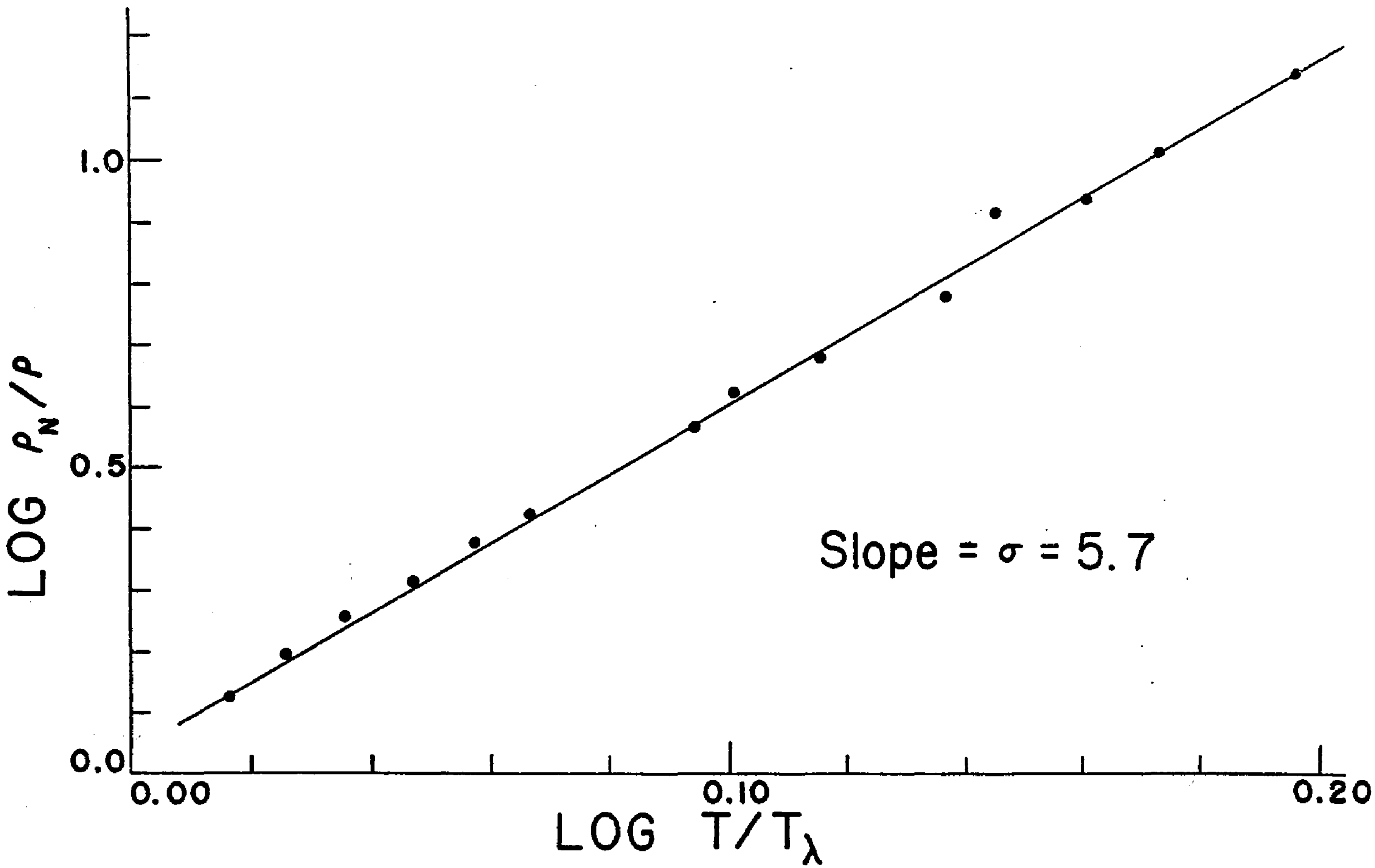


Figure 13

SELECTED BIBLIOGRAPHY

Allen, J. F. and Misener, A. D., Nature 141, 75 (1939).

Andronikashvilli, E. L., J. Phys. U.S.S.R. 10, 201 (1946).

Band, W. and Meyer, L. Phys. Rev. 74, 386 (1948).

Dash, J. G. and Taylor, R. D., Phys. Rev. 105, 7 (1957).

de Klerk, D., Hudson, R. P., and Pellam, J., Phys. Rev. 89, 662 (1953).

Hollis-Hallett, A. C., Proc. Phys. Soc. (London) A63, 1367 (1950).

Hollis-Hallett, A. C., Proc. Roy. Soc. (London) A210, 404 (1952).

Kamerlingh-Onnes, H., Proc. Acad. Sci. Amsterdam 13, 1093 (1911).

Kapitza, P. L., Nature 141, 74 (1938).

Keesom, W. H. and Wolfke, M., Leiden. Comm. 190b, (1927).

Keesom, W. H. and Wolfke, M., Proc. Acad. Sci. Amsterdam 31, 81 (1928).

Keesom, W. H. and MacWood, G., Physica 5, 737 (1938).

Landau, L., J. Phys. U.S.S.R. 5, 71 (1941).

Lee, T. D., Huang, K. and Yang, C. N., Phys. Rev. 106, 1135 (1957).

London, F., Nature 141, 643 (1938).

London, F., Phys. Rev. 54, 947 (1938).

London, F., J. Phys. Chem. 43, 49 (1939).

London, F., Revs. Modern Phys. 17, 310 (1945).

London, F., Report of International Conference of Physical Society (London) 2, 1 (1946).

Page, L. Introduction to Theoretical Physics. Third Edition. New York: D. Van Nostrand Company, Inc., 1952. p. 245.

Peshkov, V. P., J. Phys. U.S.S.R. 10, 389 (1946).

Peshkov, V. P., J. Exptl. and Theoret. Phys. U.S.S.R. 18, 951 (1948).

Prandtl, L. Essentials of Fluid Dynamics. New York: Hafner Publishing Company, 1952. p. 186.

Stokes, G. G., Trans. Com. Phil. Soc. 8, 105 (1843).

Tisza, L., Nature 141, 913 (1938).

Tisza, L., Compt. rend. (Paris) 207, 1035 (1938).

Tisza, L., J. Phys. radium 1, 165, 350 (1940).

Tisza, L., Phys, Rev. 72, 838 (19

Wilhelm, J. D., Misener, A. D. and Clark, A. R., Proc. Roy. Soc. (London) A151, 342 (1935).

APPEXDIX I

CALIBRATION OF THE COLUMN OSCILLATOR

The column oscillator was calibrated in different gasses to determine the geometrical factors A and B in the equation

$$I_p = \rho AP + \rho BP^2 \qquad (1)$$

where I_p is the moment of inertia due to the layer of the liquid medium which clings to the surfaces, ρ is the density of the medium, and P is the penetration depth. To carry out this calibration a twelve liter flask was adapted to be used on the cryostat. For all gases and pressure ranges used the expression

$$\frac{(b - a)}{P} > 4 \qquad (2)$$

was fulfilled (where b is the radius of the container, a is the radius of the oscillator and P is the penetration depth). This was necessary to reduce the effect of the container walls to less than 1% as had been shown by Dash and Taylor.[1]

With the twelve liter flask in place, the charcoal trap was attached to one of the flues of the cryostat with a thick walled vacuum tube. The charcoal trap was reactivated by heating and pumping off the liberated gas. After reaching the ultimate vacuum possible with the mechanical vacuum pump, the pump was clamped off and the charcoal trap immersed in liquid nitrogen. The vacuum obtained was determined with an ion gauge. Oscillator period and amplitude data were taken at this

---

[1]J. G. Dash and R. D. Taylor, Phys. Rev. 105, 7 (1957).

pressure, and were designated as the vacuum data. Next, one of the gases was admitted into the flask until the pressure was slightly above atmospheric pressure. The charcoal trap and ion gauge were removed. A clamped off hose was placed on one of the flues and the secondary vacuum system was attached to the other. The system was then flushed out with the gas to remove traces of air, etc. which might have been admitted during removal of the charcoal trap.

A charge of gas was left in the cryostat and after the pressure, as measured with the closed arm mercury manometer, had reached equilibrium, it and room temperature were recorded. Oscillator period and amplitude data were recorded at this pressure and temperature. Similiar data were taken at other pressures below atmospheric pressure and at room temperature.

Argon, Nitrogen and Freon 12 were the gases used in the calibration. At room temperature, i.e. 26° C, Argon and Nitrogen were above their critical temperature, therefore the values of density could be calculated from S.T.P. values. For Freon 12, however, values of density were calculated using the second virial coefficient.

Viscosity values as given in the Handbook of Chemistry and Physics[2] were extrapolated to the temperature of the experiment. They were not a function of pressure in the pressure range used.

The penetration depth, P, was calculated for each gas at the different pressures by

$$P = \sqrt{\frac{\theta \eta}{\pi \rho}} \qquad (3)$$

---

[2] Handbook of Chemistry and Physics, Chemical Rubber Publishing Co., 1950 (Thirty-Second Edition) Pp. 1835-1839.

where θ is the period of the oscillator, η is the viscosity of the medium and ρ is the density of the gas.

The periods were corrected by viscous loss by the expression:

$$\theta_c^2 = \theta^2 \left(\frac{1}{1 + \frac{\delta^2}{4\pi^2}} \right) \tag{4}$$

where θ is the measured period and δ is the logarithmic decrement of the oscillator. Values of A and B were then determined from the experimental date as follows:

$$\theta_o^2 = K\, I_o \tag{5}$$

where θ_o is the corrected period in vacuum and I_o is the moment of inertia of the oscillator and K includes the constant of the suspension.

Then

$$\theta^2 = K\,(I_o + I_p) \tag{6}$$

where θ is the period in gas and I_p is the added moment of inertia due to the clinging layer of gas.

Therefore

$$\frac{\theta^2 - \theta_o^2}{\theta_o^2} = \frac{I_p}{I_o} \quad . \tag{7}$$

Substituting (1) into (7) we obtain

$$\frac{I_o}{\rho}\left(\frac{\theta^2 - \theta_o^2}{\theta_o^2}\right) = AP + BP^2 \quad . \tag{8}$$

Setting

$$\psi = \frac{I_o}{\rho}\left(\frac{\theta^2 - \theta_o^2}{\theta_o^2}\right) \tag{9}$$

then

$$\psi = AP + BP^2 \quad . \tag{10}$$

A plot of ψ versus P and ψ/P versus P is shown in Figure I-1. The graph of ψ/P versus P is seen to be a straight line. The intercept gives the value for A and the slope the value of B. This gives a value of 200 for A and -30 for B.

Using the concept of effective inertia for the gas, similiar plots as above give a value of 180 for A and -60 for B. In either case the value of I_p due to the clinging layer of liquid Helium is not consistent with a calculated value using geometrical methods for evaluating A.

Using geometrical methods the value of A is 85, neglecting terms in P^2 and higher. This value was found by adding the moments of inertia of the individual surfaces of the column oscillator. Neglecting the BP^2 term due to small values of P in liquid Helium, the moment of inertia due to the clinging film becomes

$$I_p = \rho AP \quad . \tag{11}$$

From geometrical considerations[3] the moment of inertia due to the disk surfaces becomes

$$I_{p_d} = \rho \pi PR^4 - 4 P \rho \pi a^2 \left\{ r_c^2 + \frac{a^2}{2} \right\} \tag{12}$$

and the moment of inertia due to layers of Helium on the four columns is

$$I_{p_c} = 4 \pi \rho h Pa (r_c^2 + a^2) \quad . \tag{13}$$

The total moment of inertia is the sum of these two or

$$I_p = I_{p_d} + I_{p_c} = \rho P \left[4 \pi ha \left\{ r_c^2 + a^2 - \frac{r_c^2 a}{h} - \frac{a^3}{2h} \right\} + \pi R^4 \right] \tag{14}$$

---

[3]All symbols have been defined previously in Chapters IV and V.

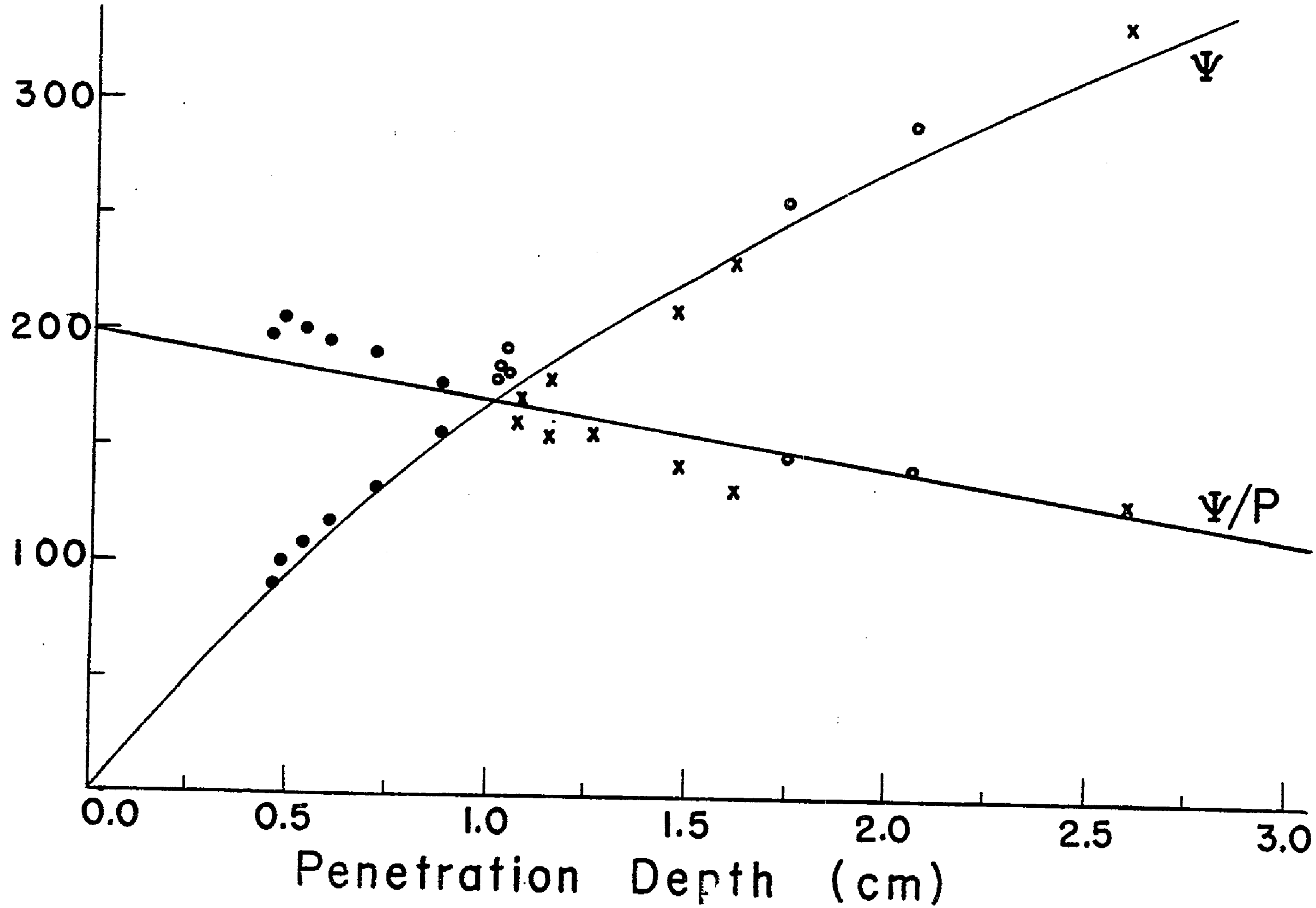


Figure I-1

Neglecting P^2 and higher order terms and substituting in corrected values of R, a, r_c and h into equation (14) gives

$$I_p = \rho P \times 85 = \rho PA$$

therefore A = 85.

The large discrepancy in the values of A obtained by the two different methods outlined above cannot be satisfactorily explained. It is possible that the data for the gas calibration were wrongly interpreted. For example, at lower pressures the penetration depth became larger than the distance between adjacent columns and in some instances greater than the distance between the disks. At such penetration depth the actual mass of gas, hence the inertia, becomes less than the predicted values and equation (1) above for I_p becomes erroneous. Also it is possible that with penetration depths less than were possible with our system the curve ψ/P versus P could have a positive slope thus intercepting the ordinate at a smaller value than that obtained.

The value of A = 85 gave the best correlation between observed and theoretical values of effective inertia. This correlation is consistent with other observations on effective mass; in particular, those made by Birkhoff[4] in an application of effective mass to the initial acceleration experienced by a spherical hydrogen balloon in air. This is correctly predicted from the effective mass concept to be 5g/3. Birkhoff further predicted that even in dense viscous liquids such as water the effective mass concept would still be valid. Therefore, it seems plausable that

---

[4]G. Birkhoff, Hydrodynamics, York, Ba.: Princeton University Press, 1950, Pp. 152-3.

the normal component of liquid Helium would obey this effective mass concept. For it to do so the best correlation requires that one choose the value of 85 for A.

Experimental data for the calibration of the column oscillator to determine the geometrical constants A and B:

$\rho \times 10^{+3}$ (g/ml)	θ^2	ψ	P (cm.)	ψ/P
Argon gas:				
1.450	528.954	193.72	1.053	183.96
1.510	529.046	186.70	1.027	181.79
0.525	516.380	257.96	1.750	147.40
0.372	514.060	291.84	2.070	140.98
Freon 12 gas:				
4.437	539.215	90.06	0.455	197.90
3.838	538.147	100.90	0.489	206.20
3.153	533.980	107.42	0.538	199.50
2.515	530.381	118.19	0.602	196.39
1.852	525.968	133.07	0.700	190.00
1.184	520.661	155.86	0.873	178.40
Nitrogen gas:				
1.107	521.300	173.71	1.080	160.84
0.963	519.749	181.05	1.160	156.07
0.801	518.382	198.03	1.270	155.92
0.633	516.153	209.93	1.470	142.80
0.457	513.793	230.99	1.670	138.20
0.188	510.127	335.30	2.600	128.90

APPENDIX II

CALIBRATION OF THE GAUZE OSCILLATOR

The gauze oscillator was calibrated in the same manner as the column oscillator. Gaseous Helium and Argon were used and the necessary data were obtained using the twelve liter flask as before. The data obtained from the two gases at room temperature and varying pressures were used as follows;

$$\theta_o^2 = K\, I_o \tag{1}$$

$$\theta^2 = K\,(\,I_o + I_v + I_p\,) \tag{2}$$

where as usual, θ_o is the corrected period obtained in vacuum, I_o is the moment of inertia in vacuum and I_v is the moment of inertia due to the mass of gas contained in the volume, V, of the gauze excluding the volume of the fibers,

$$I_v = 1/2\,\rho\, VR^2 = \rho\, V' \quad . \tag{3}$$

I_p is the moment of inertia due to the contribution of the layer of Helium on the outer surface of the gauze. As before

$$I_p = \rho\,(\,AP + BP^2) \quad . \tag{4}$$

θ is the period of the oscillator in the gas.

Combining (1) and (2) to eliminate K we obtain

$$I_o \left(\frac{\theta^2 - \theta_o^2}{\theta_o^2}\right) = I_v + I_p = \rho\,(V' + AP + BP^2) \quad . \tag{5}$$

Again defining

$$\frac{I_o}{\rho}\left(\frac{\theta^2}{\theta_o^2} - 1\right) = \psi = V' + AP + BP^2 \quad . \tag{6}$$

Data for ψ and P were plotted and are shown in Figure II-1. As can be

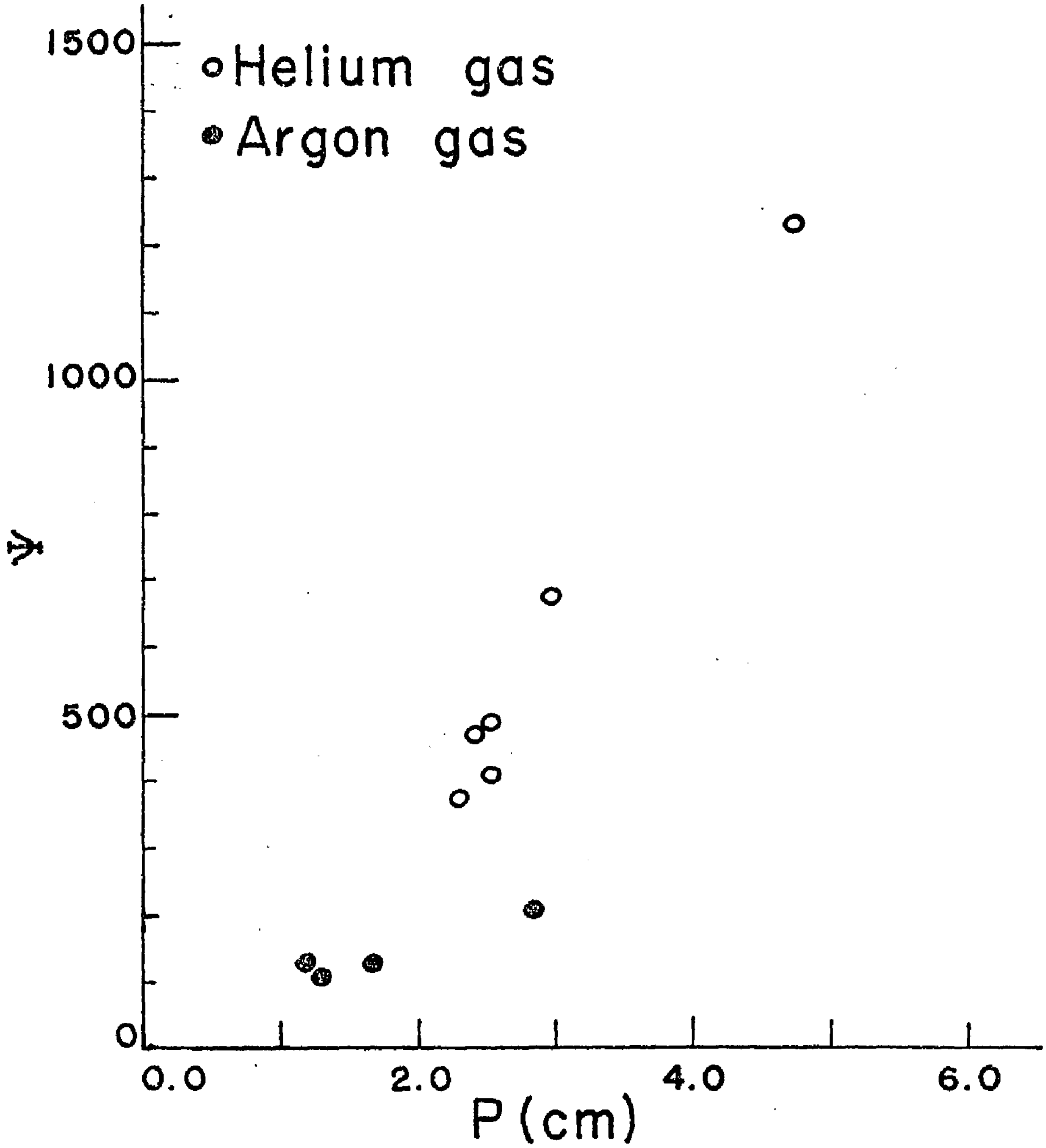


Figure II-1

seen these results are not significant. This is probably due to the necessary inclusion of the $\rho V'$ term. In a gas the $\rho V'$ term is not meaningful because of the sorbed gas on the surfaces of the fibers in the gauze oscillator and would only be significant if this factor could be accounted for. At low pressures a condition is approached where the $\rho V'$ term becomes essentially a function of this sorbed gas term only.

An attempt was made to calibrate the system in a liquid, thereby eliminating the unknown contribution of the surface effect. An examination of the available data for viscosity and density as a function of temperature virtually eliminated any of the usable liquids except water. Water at 26° C. (room temperature) was placed in the small Dewar flask. At this temperature the penetration depth was approximately 0.32 cm. Again the condition

$$\frac{(b - a)}{P} > 4$$

was met to reduce the effect of the walls to less than 1% (see Appendix I). The Dewar was used to help eliminate thermal conduction in the water. When temperature equilibrium was reached, the electronic system was set up and a run was attempted. It was found that the system was nearly critically damped and there was too much mechanical vibration in the system to obtain reliable results. More elaborate temperature control of the water plus a different suspension system would be necessary to calibrate the system in water.

A and V' were subsequently evaluated from geometrical considerations to be 39.8 and 34 respectively. These values were obtained by neglecting the BP^2 and higher order terms of P as follows. The moment of inertia due to the clinging layer on the outer surfaces is the sum of two parts, one part, I_{p_d}, is the sum of the moments of inertia of two disks of radius R and the other part, I_{p_c}, is the moment due to a cylinder of radius R and height h, where R is the radius and h is the height of the gauze. Thus

$$I_p = \rho AP = 2I_{p_d} + I_{p_c} \tag{7}$$

or

$$I_p/\rho = AP = 2\left\{1/2(P/2 \times \pi R^2)\right\} R^2 + \pi h \left\{(R + P/2)^2 - R^2\right\} R^2 . \tag{8}$$

From this

$$A = \pi R^3(R/2 + h) = 39.8 \tag{9}$$

V' was determined from the definition

$$V' = 1/2 \; V R^2 \tag{10}$$

where V was the volume of the roll of gauze minus the volume of the fibers. This volume was found to be 2/3 the actual volume V_T of the roll of gauze determined by submersion of the roll in ether in a graduated cylinder.

Therefore

$$V' = 1/2 \; V R^2 = 1/2(2/3 \; V_T) R^2 = 1/3 \; V_T R^2$$
$$= 1/3 \; \pi h \; (R^2 - r_s^2) R^2 = 34 \tag{11}$$

where r_s is the radius of the spindle that the gauze was wound onto.

Substituting these values of A and V' into the definition for X we obtain a value for X of 1.114 at 1.354°K.

$$X = \frac{V' + AP}{V' + AP_\lambda}$$

X is 1 by definition at the λ-point. Therefore, X changes by 10% in the temperature range of 1.354°K to the λ-point.

Experimental data for the calibration of the gauze oscillator to determine the geometrical constants V' and A:

$\rho \times 10^{+3}$ (g/ml)	θ^2	ψ	P (cm.)	ψ/P
Argon Gas: (Viscosity = 22.6×10^{-6} poise)				
1.60	1131.582	133.6	1.23	108.6
1.35	1126.139	111.7	1.33	83.9
0.87	1122.786	129.2	1.66	77.8
0.30	1118.768	218.6	2.83	77.2
Helium Gas: (Viscosity = 87.1×10^{-6} poise)				
0.162	1118.434	381.4	2.27	168.0
0.142	1118.903	471.1	2.43	193.5
0.128	1118.434	482.8	2.57	188.2
0.127	1117.565	405.5	2.58	157.5
0.097	1118.768	679.0	2.96	229.0
0.038	1117.164	1230.0	4.71	261.1
0.017	1116.362	2232.1	7.08	315.4

APPENDIX III

TABULATED EXPERIMENTAL AND CALCULATED DATA FOR THE COLUMN AND GAUZE OSCILLATORS IN LIQUID HELIUM

All symbols used in the following tabulations have been defined in Chapters IV and V.

Tabulated data to determine values of effective inertia for the column type oscillator in liquid Helium II:

$T^{o}K$	θ	ρ_N	ρ_s	P	δ	I_S
1.354	26.324	0.0076	0.1374	0.1488	0.080	2.138
1.485	26.411	0.0124	0.1326	0.1049	0.098	2.063
1.501	26.395	0.0150	0.1300	0.0900	0.090	2.023
1.564	26.437	0.0200	0.1250	0.0802	0.100	1.945
1.656	26.496	0.0260	0.1190	0.0676	0.122	1.852
1.870	26.669	0.0545	0.0905	0.0467	0.150	1.408
1.952	26.748	0.0680	0.0770	0.0433	0.166	1.198
2.048	26.906	0.0890	0.0560	0.0402	0.262	0.871
2.100	27.031	0.1070	0.0380	0.0400	0.270	0.591
2.143	27.235	0.1270	0.0180	0.0399	0.277	0.280
2.177	27.389	0.1450	0.0000	0.0453	0.293	0.000

Vacuum 22.475

Data comparing theoretical and experimental values of effective inertia for the column type oscillator in liquid Helium II:

$T^{o}K$	I_E(Eq. V.3)	I_E(Eq. VI)	I_p(A=200)	I_p(A=85)	I_{E_x}(A=200)	I_{E_x}(A=85)
1.354	2.3112	2.2524	0.225	0.096	2.108	2.237
1.485	2.3198	2.2596	0.260	0.111	2.126	2.275
1.501	2.3218	2.2611	0.270	0.148	2.106	2.228
1.564	2.3358	2.2504	0.320	0.136	2.082	2.265
1.656	2.3433	2.2794	0.351	0.149	2.086	2.288
1.870	2.3920	2.3159	0.309	0.216	2.233	2.326
1.952	2.4057	2.3354	0.589	0.250	2.001	2.339
2.048	2.4426	2.3658	0.715	0.304	1.964	2.374
2.100	2.4536	2.3825	0.855	0.364	1.901	2.393
2.143	2.4776	2.4378	1.012	0.431	1.869	2.452
2.177	2.5864	———	1.314	0.558	1.664	2.420

Determination of ρ_N/ρ :

T^oK	θ	$\dfrac{\theta^2 - \theta_o^2}{\theta_\lambda^2 - \theta_o^2}$	ρ_N/ρ
For gauze oscillator one:			
1.472	33.844	0.1863	0.0812
1.567	34.187	0.2280	0.1284
1.631	34.440	0.2580	0.1622
1.680	34.667	0.2860	0.1938
1.754	35.110	0.3412	0.2562
1.899	36.168	0.4750	0.4072
2.008	37.216	0.6120	0.5619
2.094	38.376	0.7670	0.7369
For gauze oscillator two:			
1.388	34.832	0.1754	0.0689
1.469	35.016	0.1980	0.0945
1.500	35.134	0.2124	0.1107
1.564	35.308	0.2184	0.1175
1.593	35.533	0.2618	0.1665
1.666	35.813	0.2969	0.2062
1.699	35.999	0.3203	0.2326
1.750	36.253	0.3524	0.2688
1.865	37.048	0.4545	0.3841
1.903	37.323	0.4903	0.4245
1.947	37.721	0.5426	0.4836
1.999	38.231	0.6105	0.5602
2.048	38.782	0.6848	0.6441
2.096	39.450	0.7763	0.7471
2.174	41.037	1.0000	1.0000

AUTOBIOGRAPHY

Billy J. Good was born May 1, 1924 in Alma, Arkansas. He attended public schools in Arkansas until 1940. After serving four years in the United States Coast Guard he entered Arkansas State Teachers College in 1948. He received the Bachelor of Science degree in physics from this institution in 1950. He entered the Graduate School at the University of Arkansas in 1950 and received the Master of Science degree in physics in 1952. He then served as an instructor in physics at this institution for two years. He entered the Graduate School at Louisiana State University in 1954 and is now a candidate for the degree of Doctor of Philosophy in the Department of Physics.

EXAMINATION AND THESIS REPORT

Candidate: Billy J. Good

Major Field: Physics

Title of Thesis: Some Hydrodynamical Aspects of Oscillating Symmetrical Bodies in Liquid Helium II

Approved:

J. M. Reynolds
Major Professor and Chairman

Richard J. Russell
Dean of the Graduate School

EXAMINING COMMITTEE:

W. C. Rasch [illegible]

Vincent E. Parker

Ralph C. Mobley

E. Schuhman

Date of Examination:

October 12, 1957